A SPECIAL INEQUALITY FOR TRIANGLE

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Abstract. In this paper we present a special inequality for triangle.

Main results:

1. Theorem:

If $u_k, v_k \in (0,\infty); n \in \mathbb{N}; n \geq 2; k \in \overline{1,n}$ then

$$u_1\left(\frac{v_2v_3}{v_1}\right)^{\lg\frac{v_2}{v_3}} + u_2\left(\frac{v_3v_1}{v_2}\right)^{\lg\frac{v_3}{v_1}} + u_3\left(\frac{v_1v_2}{v_3}\right)^{\lg\frac{v_1}{v_2}} \geq 3\sqrt[3]{u_1u_2u_3}$$

Proof.

$$\sum u_1\left(\frac{v_2v_3}{v_1}\right)^{\lg\frac{v_2}{v_3}} \geq 3\sqrt[3]{\prod u_1(\frac{v_2v_3}{v_1})^{\frac{v_2}{v_3}}} =$$

$$= 3\sqrt[3]{u_1u_2u_3} \text{ because if}$$

$$M = \prod\left(\frac{v_2v_3}{v_1}\right)^{\lg\frac{v_2}{v_3}} \text{ then:}$$

$$\lg M = \sum\left(\lg\frac{v_2}{v_3}\right)\lg\frac{v_2v_3}{v_1} =$$

$$= \sum(\lg v_2 - \lg v_3)(\lg v_2 + \lg v_3 - \lg v_1) = 0$$

therefore $M = 1$

□

Corrolaries:

In any triangle ABC:

1. $\sum r_a\left(\frac{bc}{a}\right)^{\lg\frac{b}{c}} \geq 3\sqrt[3]{S^2r}$
2. $\sum(S-a)\left(\frac{r_br_c}{r_a}\right)^{\lg\frac{r_b}{r_c}} \geq 3\sqrt[3]{Sr^2}$
3. $\sum h_a\left(\frac{\sin B\sin C}{\sin A}\right)^{\lg\frac{\sin B}{\sin C}} \geq 3\sqrt[3]{\frac{2S^2r^2}{R}}$
4. $\sum \cot\frac{A}{2}\left(\frac{\tan\frac{A}{2}\tan\frac{C}{2}}{\tan\frac{A}{2}}\right)^{\lg\frac{\tan\frac{B}{2}}{\tan\frac{C}{2}}} \geq 3\sqrt[3]{\frac{S}{r}}$
5. $\sum \tan\frac{A}{2}\left(\frac{\cot\frac{B}{2}\cot\frac{C}{2}}{\cot\frac{A}{2}}\right)^{\lg\frac{\cot\frac{B}{2}}{\cot\frac{C}{2}}} \geq 3\sqrt[3]{\frac{r}{S}}$
6. $\sum \sin^2\frac{A}{2}\left(\frac{\cos^2\frac{B}{2}\cos^2\frac{C}{2}}{\cos^2\frac{A}{2}}\right)^{\lg\frac{\cos^2\frac{B}{2}}{\cos^2\frac{C}{2}}} \geq 3\sqrt[3]{(\frac{r}{4R})^2}$
7. $\sum \cos^2\frac{A}{2}\left(\frac{\sin^2\frac{B}{2}\sin^2\frac{C}{2}}{\sin^2\frac{A}{2}}\right)^{\lg\frac{\sin^2\frac{B}{2}}{\sin^2\frac{C}{2}}} \geq 3\sqrt[3]{(\frac{s}{4R})^2}$

Key words and phrases. special inequality, triangle.

References

[1] Octogon Mathematical Magazine, 1993-2015