Another Generalization of the Sawayama's Lemma and Sawayama and Thébault's Theorem

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Abstract

In this note we give a generalization of generalization of the Sawayama's Lemma and Sawayama and Thébault's Theorem without proof.

1 A generalization of the Sawayama lemma

Problem 1. Let ABC be a triangle with the incenter I, let (O) be a circle through B, C. Let (O_A) be a circle such that (O_A) tangent to AB, AC, and tangent to (O), such that common point of $(O_A), (O)$ and A are in the same half plane divides by BC, and A, O_A, I are collinear. Let P be a point outside of (O_A), let L be a line through P and tangent to (O_A). Let (O_1) be the circle tangent to BC, and tangent to L, and (O_1) tangent to (O) such that:

1-if (O_A) externally tangent to (O), selected (O_1) and (O_A) are not in the same half plane divides by L, (see Figure 1).

2-if (O_A) internally tangent to (O), selected (O_1) and (O_A) are in the same half plane divides by L, (see Figure 2).

Let (O_1) tangent to BC at D, (O_1) tangent to L at E. Then show that D, E, I are collinear.

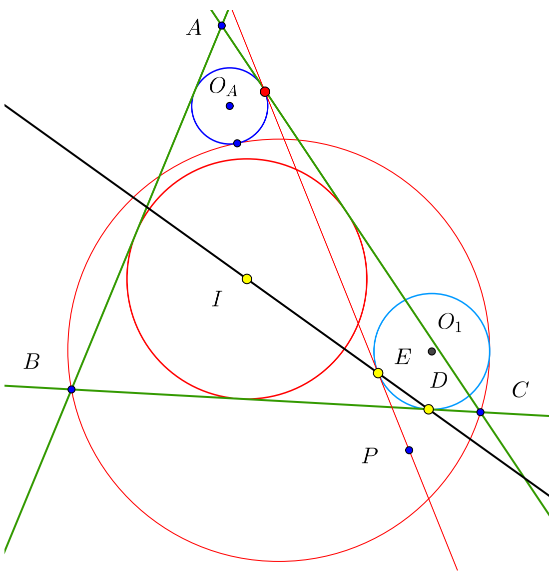

Figure 1: (O_A) externally tangent to (O)

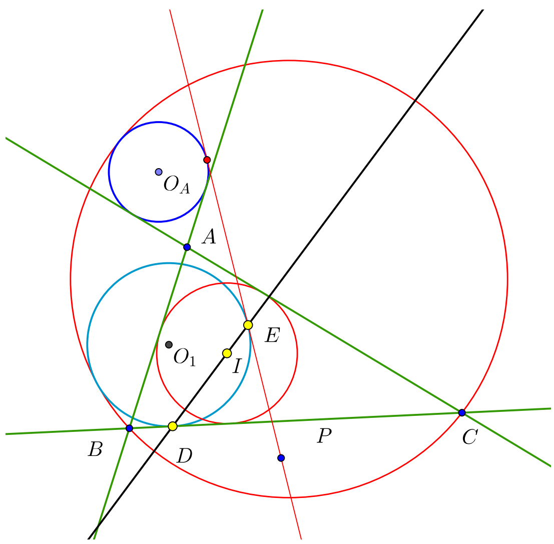

Figure 2: (O_A) internally tangent to (O)

2 A generalization of the Sawayama-Thebault theorem

Problem 2. Let ABC be a triangle with the incenter I, let (O) be a circle through B, C. Let (O_A) be a circle such that (O_A) tangent to AB, AC, and (O), such that common point of $(O_A), (O)$ and A are in the same half plane divides by BC, and A, O_A, I collinear. Let P be a point outside of (O_A), let L_1, L_2 be two lines through P and tangent to (O_A). Let $(O_1), (O_2)$ be two circles, such that (O_1) tangent to (O), (O_1) tangent to L_1 and (O_1) tangent to BC, (O_2) tangent to (O), (O_2) tangent to L_2 and (O_2) tangent to BC.

1. If (O_A) externally tangent to (O), selected $(O_1), (O_2)$, such that (O_1) and (O_A) are not in the same half plane divides by L_1, (O_2) and (O_A) are not the same half plane divides by L_2 (Figure 3).

2. If (O_A) internally tangent to (O), selected $(O_1), (O_2)$ such that (O_1) and (O_A) are the same half plane divides by L_1, (O_2) and (O_A) are the same half plane divides by L_2 (Figure 4).

Then show that the line O_1O_2 through a fixed point when P move on a given line.

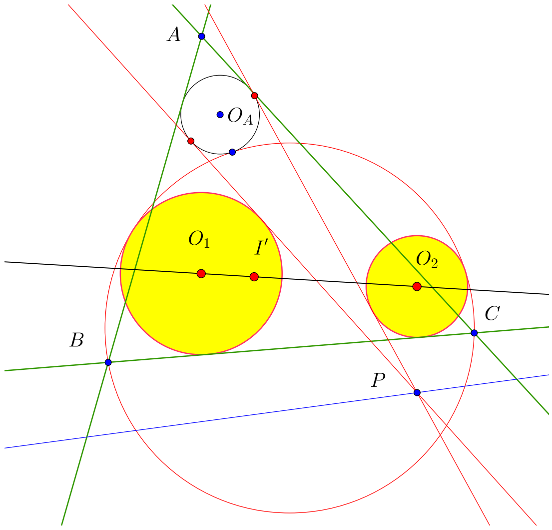

Figure 3: (O_A) externally tangent to (O)

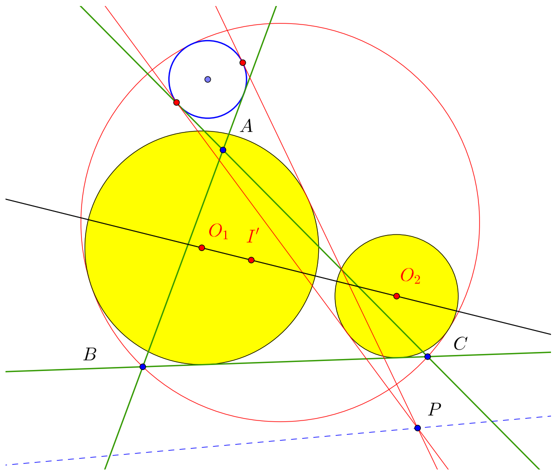

Figure 4: (O_A) internally tangent to (O)

3 Variants

There are many variants of problem 1 and problem 2. Example:

Problem 3. Let ABC be a triangle with the incenter I, and excenter E_A, let (O) be a circle through B, C. Let (O_A) be a circle such that (O_A) tangent to AB, AC, and (O_A) internally (or externally) tangent to (O). Let common point of two circles $(O_A), (O)$ and A are not in the same half plane divides by BC. Let P be a point outside of (O_A), let L be a line through P and tangent to (O_A). Let (O_1) be the circle such that (O_1) and (O_A) are in the same half plane divides by L, and O_1 tangent to BC, and tangent to L. Let (O_1) tangent with BC at D, (O_1) tangent L at E. Then show that

1-D, E, I are collinear if (O_A) internally tangent to (O) (Figure 5).

2-D, E, E_A are collinear (O_A) externally tangent to (O) (Figure 6).

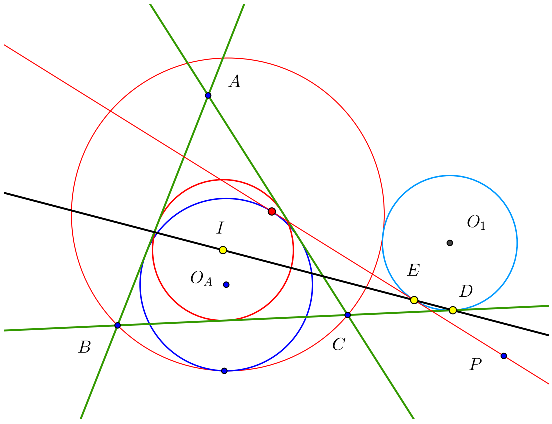

Figure 5: (O_A) externally tangent to (O)

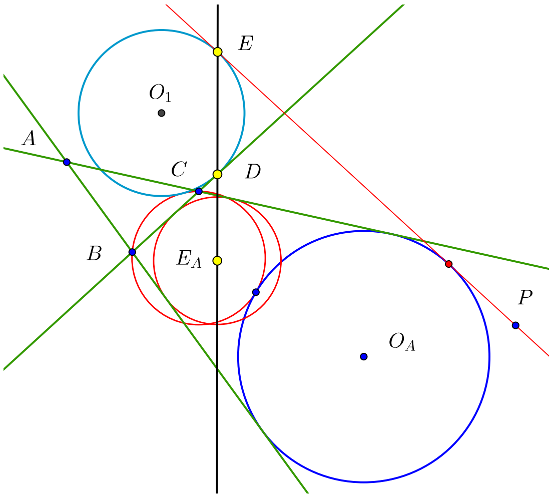

Figure 6: (O_A) internally tangent to (O)

References

[1] Jean-Louis Ayme, *Sawayama and Thébault's Theorem*, Forum Geometricorum 3 (2003) 225–229.

[2] Dao Thanh Oai, *A Generalization of Sawayama and Thébault's Theorem*, International Journal of Computer Discovered Mathematics, Volume 1 Number 3 (September 2016) pp.33-35.

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