Properties Of Parabola Inscribed In a Triangle

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Abstract. In this work we present new theorems on Artzt parabolas, using barycentric coordinates.

Keywords. geometry, Artzt Parabolas, collinearity.

1. Introduction

Consider a conic which is tangent at B, C to the sides AB, AC. We call this an $A-$conic. The A-conic is a parabola if the center $(p : f : f)$ is on the line at infinity. Hence, $p + f + f = 0$, and the A-parabola has equation $x^2 - 4yz = 0$. We label this parabola $\mathcal{P}_a$. Similarly, the $B-$ and $C-$parabolas are $\mathcal{P}_b$: $y^2 - 4zx = 0$ and $\mathcal{P}_c$: $z^2 - 4xy = 0$ respectively. These are also known as the Artzt parabolas [3].

Some properties of these parabolas are mentioned in [1] and [2].

2. Additional Properties

Theorem 2.1. *Let ABC be a triangle with side lengths are a, b, c and $\mathcal{P}_a$ be a parabola whose focus is F and vertex is P. $\mathcal{P}_a$ is tangent to sides AB and AC at B and C respectively. Let K be symmedian point of ABC. A, K and F are collinear.(see Fig. 1)*

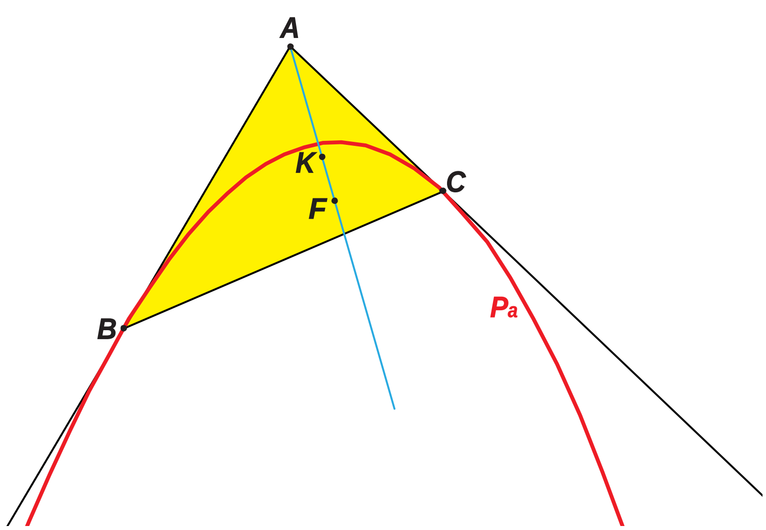

Figure 1.

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Proof. In homogenous barycentric coordinates respect to triangle ABC, $\mathcal{P}_a$ has equation $x^2 - 4yz = 0$ and its focus coordinates are $\{a^2 - b^2 - c^2 : -b^2 : -c^2\}$. $A = \{1 : 0 : 0\}$ and $K = \{a^2 : b^2 : c^2\}$. Hence determinant of these points is 0, they are collinear. □

Theorem 2.2. *Let O be circumcircle center of ABC and d is directrix of $\mathcal{P}_a$. d, OF and BC are concurrent lines at a point X. (see Fig. 2)*

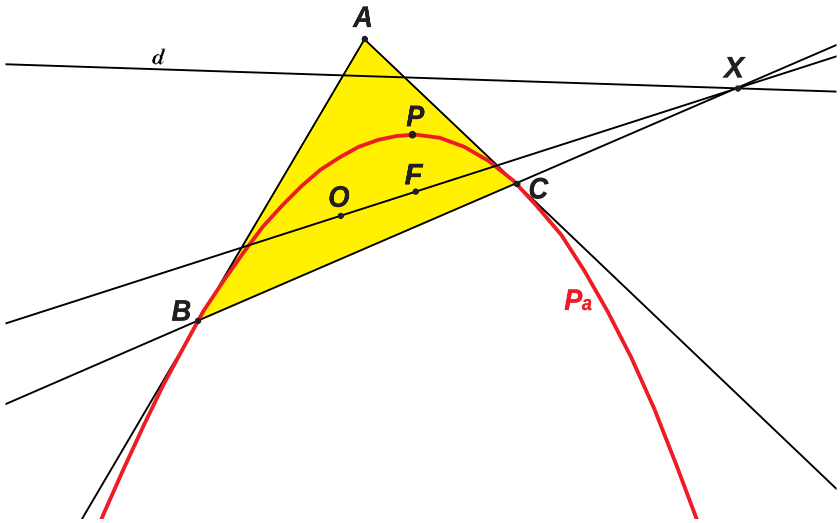

FIGURE 2.

Proof. BC has equation $y - z = 0$ and OF has equation;

$$2b^2c^2x - c^2(-a^2 + b^2 + c^2)y - b^2(-a^2 + b^2 + c^2)z = 0$$

and the intersection point of BC and OF is $(0 : -b^2 : c^2)$. Equation of directrix is;

$$(a^2 - b^2 - c^2)x + 2c^2y + 2b^2z = 0$$

The intersection point of BC and d is $(0 : -b^2 : c^2)$. Hence d, BC and OF are concurrent at point $(0 : -b^2 : c^2)$. □

Theorem 2.3. *Let Q be the point of intersection of median AM with directrix of $\mathcal{P}_a$. H is orthocenter of ABC. Q, H and vertex P are collinear. (see Fig. 3)*

Proof. In homogenous barycentric coordinates respect to triangle ABC, Q has coordinates,

$$\left\{2\left(b^2 + c^2\right) : -a^2 + b^2 + c^2 : -a^2 + b^2 + c^2\right\}$$

Vertex P has coordinates,

$$\left\{2(a^2 - b^2 - 3c^2)(a^2 - 3b^2 - c^2) : (-a^2 + 3b^2 + c^2)^2 : (-a^2 + b^2 + 3c^2)^2\right\}$$

and the coordinate of H is,

$$\left\{a^4 - \left(b^2 - c^2\right)^2 : -a^4 + b^4 + 2a^2c^2 - c^4 : -a^4 + 2a^2b^2 - b^4 + c^4\right\}$$

Hence determinant of these points is 0, they are collinear. □

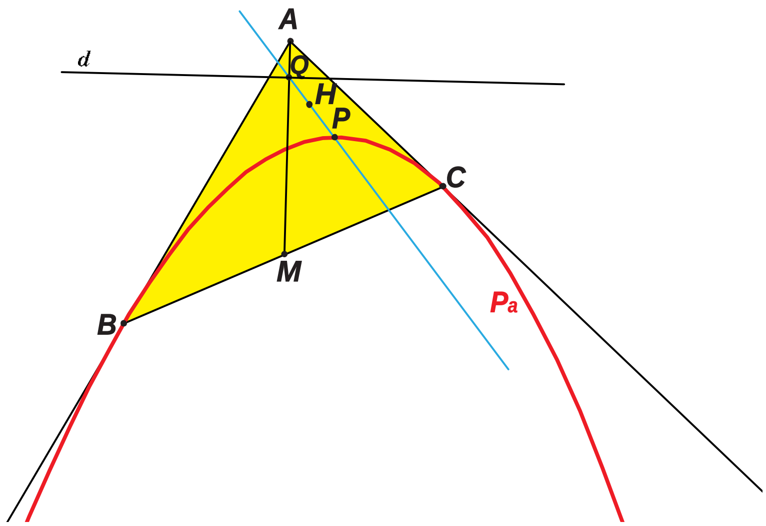

FIGURE 3.

REFERENCES

[1] J.A.Bullard, *Further Properties of Inscribed Parabola In a Triangle*, American Mathematical Monthly, Vol 44, No.6, 1937.

[2] J.A.Bullard, *Properties of Inscribed Parabola In a Triangle*, American Mathematical Monthly, Vol 42, No.10, 1935.

[3] N.Dergiades, *Conics Tangent at the Vertices to Two Sides of a Triangle*, Forum Geometricorum, Vol 10, 41-53, 2010.