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INEQUALITY IN TRIANGLE 305
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1. In ΔABC

$$\frac{R}{2r}+\frac{3p^2}{(4R+r)^2}\geq 2$$

Proposed by Adil Abdullayev - Baku - Azerbaidian

Proof.

Using Gerretsen's inequality $p^2\geq 16Rr-5r^2$, it's enough to prove that:

$$\frac{R}{2r}+\frac{3(16Rr-5r^2)}{(4R+r)^2}\geq\Leftrightarrow 16R^3-56R^2r+65Rr^2-34Rr^2\geq 0\Leftrightarrow$$

$\Leftrightarrow (R-2r)(16R^2-24Rr+17r^2)\geq 0,$ *obviously from Euler's inequality* $R\geq 2r$.

The equality holds if and only if the triangle is equilateral.

□

Remark

The inequality can be developed:

2. In ΔABC

$$n\cdot\frac{R}{r}+k\cdot\frac{p^2}{(4R+r)^2}\geq 2n+\frac{k}{3},\ \text{where } 15n\geq 2k\geq 0.$$

Proposed by Marin Chirciu - Romania

Proof.

Using Gerretsen's inequality $p^2\geq 16Rr-5r^2$ and the conditions $n\geq 0, k\geq 0$ it's enough to prove that:

$$n\cdot\frac{R}{r}+k\cdot\frac{16Rr-5r^2}{(4R+r)^2}\geq 2n+\frac{k}{3}\Leftrightarrow$$

$$48nR^3-(72n+16k)R^2r+(40k-45n)Rr^2-(6n+18k)r^3\geq 0$$

$\Leftrightarrow (R-2r)(48nR^2+(24n-16k)Rr+(3n+8k)r^2)\geq 0,$ *obviously from Euler's inequality* $R\geq 2r$ *and the observation that* $48nR^2+(24n-16k)Rr+(3n+8k)r^2\geq 0$ *for* $15n\geq 2k\geq 0$.

The equality holds if and only if the triangle is equilateral or $n=k=0$.

□

Remark

The inequality can be reformulated:

3. In ΔABC

$$\boldsymbol{\frac{R}{r}+\lambda\cdot\frac{p^2}{(4R+r)^2}\geq 2+\frac{\lambda}{3}}, \textbf{ where } \boldsymbol{0\leq\lambda\leq\frac{15}{2}}.$$

Proof.

*In **2.** we divide with n and we denote $\frac{k}{n}=\lambda$.*

The equality holds if and only if the triangle is equilateral.

*For $\lambda=6$ we obtain inequality **1.***

□

Remark

The inequality can be reformulated:

4. In ΔABC

$$\boldsymbol{\lambda\cdot\frac{R}{r}+\frac{p^2}{(4R+r)^2}\geq 2\lambda+\frac{1}{3},\lambda\geq\frac{2}{15}}.$$

Proof.

*In **2.** we divide with k and we denote $\frac{n}{k}=\lambda$.*

The equality holds if and only if the triangle is equilateral.

*For $\lambda=\frac{1}{6}$ we obtain inequality **1.***

□

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