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1. In ΔABC

$$m_a \cos\frac{A}{2} + m_b \cos\frac{B}{2} + m_c \cos\frac{C}{2} \geq \frac{9r\sqrt{3}}{2}$$

Proposed by Kevin Soto Palacios - Huarmey - Peru

Remark.

The inequality can be strengthened:

2. In ΔABC

$$m_a \cos\frac{A}{2} + m_b \cos\frac{B}{2} + m_c \cos\frac{C}{2} \geq \frac{3p}{2}$$

Proposed by Marin Chirciu - Romania

Proof.

We use the remarkable inequality $m_a \geq \frac{b+c}{2}\cos\frac{A}{2}$

We obtain:

$$\sum m_a \cos\frac{A}{2} \geq \sum m_a \cos^2\frac{A}{2} = \sum \frac{b+c}{2}\cdot\frac{p(p-a)}{bc} = \frac{p}{2}\cdot\frac{a(b+c)(p-a)}{abc} =$$

$$= \frac{p}{2}\cdot\frac{12pRr}{4pRr} = \frac{3p}{2}.$$

The equality holds if and only if the triangle is equilateral.

□

Remark.

*Inequality **2.** is stronger then Inequality **1.**:*

3. In ΔABC

$$m_a \cos\frac{A}{2} + m_b \cos\frac{B}{2} + m_c \cos\frac{C}{2} \geq \frac{3p}{2} \geq \frac{9r\sqrt{3}}{2}.$$

Proof.

*See inequality **2.** and Mitrinović's inequality:* $p \geq 3r\sqrt{3}$.

The equality holds if and only if the triangle is equilateral.

□

In the same mode can be proposed:

4. In ΔABC

$$\boldsymbol{m_a \sin\frac{A}{2} + m_b \sin\frac{B}{2} + m_c \sin\frac{C}{2} \geq \frac{ab+bc+ca}{4R}}$$

Proof.

Using the remarkable inequality $m_a \geq \frac{b+c}{2}\cos\frac{A}{2}$, *we obtain:*

$$\sum m_a \sin\frac{A}{2} \geq \sum \frac{b+c}{2}\cos\frac{A}{2}\sin\frac{A}{2} = \frac{1}{4}\sum(b+c)\sin A = \frac{1}{4}\sum(b+c)\cdot\frac{a}{2R} = \frac{ab+bc+ca}{4R}.$$

The equality holds if and only if the triangle is equilateral.

□

5. In ΔABC

$$\boldsymbol{m_a \sin\frac{A}{2} + m_b \sin\frac{B}{2} + m_c \sin\frac{C}{2} \geq \frac{S\sqrt{3}}{R}}$$

Proof.

See 4. and the remarkable inequality $ab+bc+ca \geq 4S\sqrt{3}$.

The equality holds if and only if the triangle is equilateral.

□

6. In ΔABC

$$\boldsymbol{m_a \sin\frac{A}{2} + m_b \sin\frac{B}{2} + m_c \sin\frac{C}{2} \geq \frac{r(5R-r)}{R}}$$

Proof.

See 4., the identity $ab+bc+ca = p^2+r^2+4Rr$ *and Gerretsen's inequality*

$$p^2 \geq 16Rr - 5r.$$

The equality holds if and only if the triangle is equilateral.

□

The following inequalities can be written:

7. In ΔABC

$$\boldsymbol{m_a \sin\frac{A}{2} + m_b \sin\frac{B}{2} + m_c \sin\frac{C}{2} \geq \frac{p^2+r^2+4Rr}{4R} \geq \frac{r(5R-r)}{R} \geq \frac{S\sqrt{3}}{R} \geq \frac{9r^2}{R}}$$

Proposed by Marin Chirciu - Romania

Proof.
See ***4.*** *the identity* $ab + bc + ca = p^2 + r^2 + 4Rr$, *Gerretsen's inequality* $p^2 \geq 16Rr - 5r$, *Doucet's inequality* $4R + r \geq p\sqrt{3}$ *and Mitrinović's inequality:* $p \geq 3r\sqrt{3}$.

The equality holds if and only if the triangle is equilateral.

□

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