An inequality,corollary and related theorem In tangential quadrilateral.

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Introduction

we present a new inequality with corollary and new related theorem in tangential.

We know that the contact quadrilateral is cyclic certainly because from his definition his vertex are the points in which the incircle is tangent to the four sides of the tangential so to prove that it is bicentric we must prove that it's tangential only.

Notations

Let ABCD be a tangential quadrilateral with sides h o w v area(k)-semiperimeter(s)-inradius(r)-tangent lengths(z-x-t-g) –incenter(i) -tangency chords (p) (q)-the angle between the two tangency chords (α)

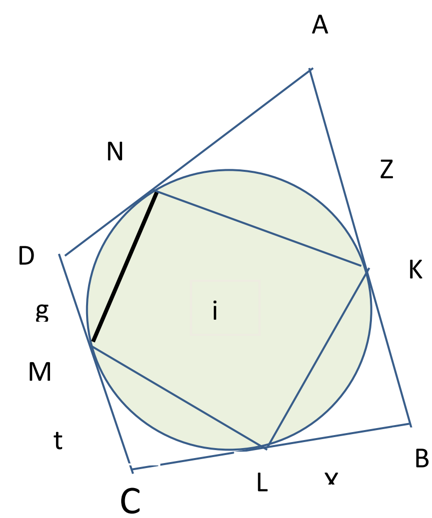
KLMN its contact quadrilateral with area(f)-sides (a-b-c-d)

Theorem 1

If KLMN is bicentric then

$K \geq 2f$

Proof by proposer



- $k \geq 4r^2$ (A-well-known inequality)
- $16r^2f^2=(ac+bd)(ad+bc)(ab+cd)$ **(parameshvara)**

BY AM-GM inequality

$4kf^2 \geq (ac+bd)(ad+bc)(ab+cd) \geq 8abcd\sqrt{(abcd)}$

- KLMN is bicentric so $4kf^2 \geq 8f.f^2$
- $k \geq 2f$

Equality holds if and only if ABCD is a square.

Corollary

In tangential quadrilateral ABCD if its contact quadrilateral is bicentric then

$$s \geq 2\sqrt{(pq)}$$

Proof by proposer

$k \geq 2f$, then $\sqrt{(howv)}.\sin((A+C)/2) \geq pq.\sin\alpha$

then $\sqrt{(howv)} \geq pq$ (since $\sin((A+C)/2) = \sin\alpha$)

BY AM-GM inequality

$(H+o+w+v) \geq 4\sqrt{(pq)}$ then $s \geq 2\sqrt{(pq)}$

Equality holds if and only if ABCD is a square.

Now we will prove a necessary and sufficient condition for the contact quadrilateral to be bicentric.

Theorem 2

In tangential quadrilateral ABCD its contact quadrilateral KLMN is bicentric if and only if the angles of the tangential ABCD satisfies the relation

$\cos(A/2)+\cos(C/2)=\cos(B/2)+\cos(D/2)$

(A-C opposite angles)

Proof by proposer

$\cos(A/2)+\cos(C/2)=\cos(B/2)+\cos(D/2)$

$\leftrightarrow rz/Ai + rt/Ci = rx/Bi + rg/Di$

$\leftrightarrow z\sin(A/2)+t\sin(C/2)=x\sin(B/2)+g\sin(D/2)$ ($\cos\beta=1-2\sin^2\beta/2$)

$\leftrightarrow \sqrt{(2z^2-2z^2\cos A)} + \sqrt{(2t^2-2t^2\cos C)} = \sqrt{(2x^2-2x^2\cos B)} + \sqrt{(2g^2-2g^2\cos D)}$ (using law of cosines)

$\leftrightarrow a+c=b+d$ (pitot theorem)

Which completes our proof.