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DOUBLE GENERALIZATION FOR TSINTSIFAS' INEQUALITY AND BĂTINEȚU'S INEQUALITY

If $m,n,p,u,v,w,x,y,z>0, m+x\ge tu, n+y\ge tu, p+z\ge tu, x+y+z\le tv, 2u\ge v$

then in $\Delta ABC, F$ – area, the following relationship holds:

$$\frac{m}{x}\cdot a^2+\frac{n}{y}\cdot b^2+\frac{p}{z}\cdot c^2\ge\frac{4(3u-v)\sqrt{3}}{v}\cdot F$$

Proposed by D.M. Bătinețu-Giurgiu and Daniel Sitaru – Romania

Solution by Marian Ursărescu – Romania

$$\frac{m}{x}a^2+\frac{n}{y}b^2+\frac{p}{z}c^2=\frac{m}{x}a^2+\frac{n}{y}b^2+\frac{p}{z}c^2+a^2+b^2+c^2-a^2-b^2-c^2=$$

$$=\frac{m+x}{x}a^2+\frac{n+y}{y}b^2+\frac{p+z}{z}c^2-\sum a^2\ge tu\left(\frac{a^2}{x}+\frac{b^2}{y}+\frac{c^2}{z}\right)-\sum a^2\quad (1)$$

From Bergström's inequality: $\frac{a^2}{x}+\frac{b^2}{y}+\frac{c^2}{z}\ge\frac{(a+b+c)^2}{x+y+z}=\frac{4s^2}{x+y+t}\ge\frac{4s^2}{4v}$ *(2)*

and $\sum a^2=2(s^2-r^2-4Rr)$ *(3)*

From (1)+(2)+(3)$\Rightarrow\frac{m}{x}a^2+\frac{n}{y}b^2+\frac{p}{z}c^2\ge\frac{u}{v}\cdot 4s^2-2(s^2-r^2-4Rr)\Rightarrow$

we must show $\frac{u}{v}4s^2-2(s^2-r^2-4Rr)\ge\frac{4\sqrt{3}(3u-v)}{v}F\Leftrightarrow$

$$\Leftrightarrow 4us^2-2v(s^2-r^2-4Rr)\ge 12\sqrt{3}uF-4\sqrt{3}vF\Leftrightarrow$$

$$\left.\begin{array}{c}4u\left(s^2-3\sqrt{3}F\right)\ge 2v\left(s^2-r^2-4Rr-2\sqrt{3}F\right)\\ But\ 2u\ge v\Leftrightarrow 4u\ge 2v\end{array}\right\}\Rightarrow \text{we must show:}$$

$$s^2-3\sqrt{3}F\ge s^2-r^2-4Rr-2\sqrt{3}F\Leftrightarrow$$

$\Leftrightarrow r^2+4Rr\ge\sqrt{3}F\Leftrightarrow r(4R+r)\ge\sqrt{3}sr\Leftrightarrow 4R+r\ge\sqrt{3}s$, *true because it is Doucet's inequality.*