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ABOUT AN INEQUALITY BY HAXVERDIYEV TAVERDI-II

By Marin Chirciu-Romania

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1) In ΔABC the following relationship holds:

$$\frac{\sqrt{m_a^2+m_b^2+m_c^2}}{\sum acosA}\geq\frac{R}{2r}$$

Proposed by Haxverdiyev Taverdi-Azerbaijan

Solution by Marin Chirciu-Romania

Using inequality: $m_a\geq\sqrt{s(s-a)}\Rightarrow m_a^2\geq s(s-a)\Rightarrow$

$$\sum m_a^2\geq\sum s(s-a)=s^2\Rightarrow\sqrt{m_a^2+m_b^2+m_c^2}\geq s;(1)$$

We have: $\sum acosA=\frac{2rs}{R};(2)$

From (1),(2) it follows that: $LHS=\frac{\sqrt{m_a^2+m_b^2+m_c^2}}{\sum acosA}\geq\frac{s}{\frac{2rs}{R}}=\frac{R}{2r}=RHD.$

Equality holds if and only if triangle is equilateral. Remark. In same class of problems.

2) In ΔABC the following relationship holds:

$$\frac{\sqrt{w_a^2+w_b^2+w_c^2}}{\sum acosA}\leq\frac{R}{2r}$$

Proposed by Marin Chirciu-Romania

Solution by proposer

Using inequality: $w_a\leq\sqrt{s(s-a)}\Rightarrow w_a^2\leq s(s-a)\Rightarrow$

$$\sum w_a^2\leq\sum s(s-a)=s^2\Rightarrow\sqrt{w_a^2+w_b^2+w_c^2}\leq s;(1)$$

We have: $\sum acosA=\frac{2rs}{R};(2)$

From (1),(2) it follows that: $LHS=\frac{\sqrt{w_a^2+w_b^2+w_c^2}}{\sum acosA}\ge\frac{s}{\frac{2rs}{R}}=\frac{R}{2r}=RHD.$

Equality holds if and only if triangle is equilateral.

3) In ΔABC the following relationship holds:

$$\frac{\sqrt{h_a^2+h_b^2+h_c^2}}{\sum acosA}\le\frac{R}{2r}$$

Proposed by Marin Chirciu-Romania

Solution by proposer

Using inequality: $h_a\le w_a\le\sqrt{s(s-a)}\Rightarrow h_a^2\le s(s-a)\Rightarrow$

$$\sum h_a^2\le\sum s(s-a)=s^2\Rightarrow\sqrt{w_a^2+w_b^2+w_c^2}\le s;(1)$$

We have: $\sum acosA=\frac{2rs}{R};(2)$

From (1),(2) it follows that: $LHS=\frac{\sqrt{h_a^2+h_b^2+h_c^2}}{\sum acosA}\ge\frac{s}{\frac{2rs}{R}}=\frac{R}{2r}=RHD.$

Equality holds if and only if triangle is equilateral.

4) In ΔABC the following relationship holds:

$$\frac{\sqrt{r_a^2+r_b^2+r_c^2}}{\sum acosA}\ge\frac{R}{2r}$$

Proposed by Marin Chirciu-Romania

Solution by proposer

We have: $r_a^2+r_b^2+r_c^2=(4R+r)^2-2s^2\overset{Doucet}{\ge}s^2\Leftrightarrow(4R+r)^2\ge3s^2$, which follows from $s^2\le4R^2+4Rr+3r^2(Gerretsen)$ and $R\ge2r(Euler)$.

So, $\sqrt{r_a^2+r_b^2+r_c^2}\ge s;(1)$

We have: $\sum acosA=\frac{2rs}{R};(2)$

From (1),(2) it follows that: $LHS = \frac{\sqrt{r_a^2+r_b^2+r_c^2}}{\sum acosA} \ge \frac{s}{\frac{2rs}{R}} = \frac{R}{2r} = RHD$.

Equality holds if and only if triangle is equilateral.

5) In ΔABC the following relationship holds:

$$\frac{\sqrt{3(a^2+b^2+c^2)}}{\sum acosA} \ge \frac{R}{r}$$

Proposed by Marin Chirciu-Romania

Solution by proposer

Using identity: $\sum a^2 = 2(s^2 - r^2 - 4Rr)$ and $\sum acosA = \frac{2rs}{R}$ inequality becomes:

$$\frac{\sqrt{6(s^2-r^2-4Rr)}}{\frac{2rs}{R}} \ge \frac{R}{r} \Leftrightarrow \sqrt{6(s^2-r^2-4Rr)} \ge 2s \Leftrightarrow$$

$6(s^2 - r^2 - 4Rr) \ge 4s^2 \Leftrightarrow s^2 \ge 3r(4R + r)$ which follows from

$s^2 \ge 16Rr - 5r^2(Gerretsen)$ and $R \ge 2r(Euler)$.

Equality holds if and only if triangle is equilateral.

Reference:

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