A TRIBUTE TO TRAIAN LALESCU

AN OUTSTANDING ROMANIAN GREAT SCIENTIST

By D.M.Bătinețu-Giurgiu,Neculai Stanciu-Romania

(Born on July 12, 1882 – Died on June 15, 1929)

Traian Lalescu's Problem – Published in Romanian Mathematical Gazette, Vol. VI, 1900-1901, as problem 579, p. 148.

Problem 579. Compute the limit:

$$\lim_{n\to\infty}\left(\sqrt[n+1]{(n+1)!}-\sqrt[n]{n!}\right)$$

Solution:

$$\lim_{n\to\infty}\left(\sqrt[n+1]{(n+1)!}-\sqrt[n]{n}\right)\overset{k!\cong\left(\frac{k}{e}\right)^k\Rightarrow\sqrt[k]{k!}\cong\frac{k}{e}}{=}\lim_{n\to\infty}\left(\frac{n+1}{e}-\frac{n}{e}\right)=\frac{1}{e}$$

Traian Lalescu started in this world from Bucharest on July 12, 1882 and there he would find his premature end on June 15, 1929.

His father, a modest bank clerk, had the same first name, Traian and was originally from Cornea commune, Caraş-Severin county. In 1876 he wrote a paper on the economic problems of agriculture and another, "The agenda of popular banks and the Lalescu coefficient method", which can be found at the Library of the Romanian Academy. His mother was from the Transylvanian side. The scientist presented himself as a native of the village near Caransebeș.

He begins his primary education in his native Bucharest. The first two middle school classes in Craiova. He took the next two middle school classes in Moldova, at Roman, then studied high school at the Boarding School in Iaşi. His name is inscribed on the high school honor roll. Traian Lalescu has always had the imprint of the environment so varied in which he was formed due to the childhood pilgrimages he made with his family: he was as solid as Banat, talkative as Olten, serious as Transylvanian, beautiful lover as Moldovan and with the sprinting spirit like the one from Bucharest.

Throughout his studies, Traian Lalescu was the first prize winner of the class and the honorary prize winner of the school, becoming from the 10th grade correspondent of the Mathematical Gazette.

In 1900 he was the first to enter the National School of Bridges and Roads in Bucharest. In the first year of studies, he is financially assisted by professor Andrei Ioachimescu, who took him home and treated him like his own child for a year.

In 1901, he published the first original mathematical note of the Mathematical Gazette, ''On a Summary of Series''.

In 1903 he retired and went to the Faculty of Sciences of the University of Bucharest, Department of Mathematics.

In 1905 he became a member of the Mathematical Gazette editorial office.

On June 17, 1905, he obtained a degree in mathematics with the grade "very good". Also in 1905 he obtained by competition, again succeeding the first, an "Adamachi" scholarship for further studies in Paris, Sorbonne, where he again obtained his Licence of Mathematics. Here he is also helped financially by professor Ion Ionescu-Bizeţ.

Between 1906 and 1910 he was a mathematics teacher at the gymnasium in Giurgiu. In 1906 he was attracted to Émile Picard's course of integral equations at the Sorbonne.

In 1907 he published four notes in Comptes Rendus des Séances de l'Academie des Sciences de Paris (CRASP).

In 1908 he defended his doctoral dissertation "Sur l'équation de Volterra", under the direction of Émile Picard, which he published both in the Gauthier-Villars Publishing House and in the prestigious journal, Journal de Mathématiques Pures et Appliquèes, Paris. Also in 1908 he published a work on Galois Theory. Thanks to the help provided by the

Romanian Academy, he presented his results at the International Congress of Mathematicians in Rome, April 6-11, 1908. Here he met Vito Volterra. The results obtained are also presented in Romania, in the Bulletin of the Société des Sciences, Bucharest (BSS).

From the summer of 1908 to the spring of 1909, he was in another large mathematical center in Göttingen, where David Hilbert and the school he had set up were located. He attended the classes of D. Hilbert and presented a paper at the Mathematical Society of Göttingen, in a meeting chaired by Felix Klein, and on June 15, 1909 he obtained the scientific title of docent.

He made his debut as university professor on June 1, 1909, as an assistant for graphic works of Ion Ionescu-Bizeţ professor. He stayed here until May 15, 1910.

After a brief return to the country, he returned to Göttingen for 1910-1911, where he gave a series of papers on his own research, which were appreciated by David Hilbert, Erhardt Schmidt and Felix Klein. Then, he goes again to Paris, where he publishes three other articles in CRASP and in our country in BSS.

Between 1910 and 1913 he was an associate professor of higher algebra at the University of Bucharest.

Between 1911 and 1912 he was transferred from Giurgiu to Bucharest, to the Central Seminary, then to the Şincai and Dimitrie Cantemir Gymnasiums.

In 1911 he published the world's first significant monograph, before Hilbert, on integral equations (the following year it was translated into French). This monograph was translated and edited in 1918, in Polish by S. Mazurkiewicz at the Polish Academy of Sciences and Letters, and as Hugo Steinhauss said, this was the book from which Polish mathematicians learned the theory of integral equations. This book was then republished by the Romanian Academy Publishing House in 1957. Vito Volterra and Édouard Goursat emphasized in their books the importance of Traian Lalescu's research on integral equations. The echoes about Traian Lalescu's works, about the results obtained in the theory of integral equations continued long after his physical disappearance. In particular, Prof. Albrecht Pietsch from Jena, in 1980, during a visit to the Institute of the Romanian Academy told to Prof. Nicolae Popa that Traian Lalsecu, together with Şerban Gheorghiu, were the first which prove that the product of two Hilbert operators - Schmidt is a track operator.

In 1911 he was appointed full professor at the School of Roads and Bridges, at the department of analytical geometry, in place of Spiru Haret and also in 1911 he was professor of rational mechanics at the University of Bucharest.

Since 1912 he has been an assistant at the department of descriptive geometry at the University of Bucharest.

After the publication of the last issue of the 21st year of the Mathematical Gazette, the First World War begins. From the following year, only the first two issues appeared in Bucharest, the occupation of the city by German troops and the destruction of the printing

house make it impossible for the magazine to appear. In December 1917, at the residence of Traian Lalescu from Iassy, it was decided to print the magazine in the capital of Moldova, at the printing house „H. Goldner" - where most of the workers were old and infirm. In order to stimulate them to print the Gazette, T. Lalescu and V. Teodoreanu brought them food from their own rations!

The number of pages per issue decreases and major dysfunctions appear in the publication of the magazine: the December 1916 issue appears in April 1917, and no. 3 of vol. XXII appears at the end of the war! And the content of the articles is different. Articles on ballistics or applications of mathematics in the military sciences are written. Number 1 of vol. XXIII is opened by the vibrant article "To the Romanian soldiers" and is dedicated to the soldiers in the front line (Gazeta had the authorization to be distributed on the front). The editorial meetings are held regularly, under the chairmanship of the venerable professor C. Climescu, the initiator of Scientific Recreations. The construction of a Mathematical Gazette House has been planned since from 1920. N. Nicolescu donates the first 500 lei for this purpose. Three years later, Traian Lalescu proposed to Tancred Constantinescu, then General Manager of the Railways, to donate a plot of land near the North Station for the construction of the place. Started in September 1933, it was completed in August 1934, and on January 27, 1935, on a Sunday, it was inaugurated. All four "pillars" of the Gazette are present. "Of all the problems proposed in the Mathematical Gazette, none was more difficult, more beautiful and more interesting than the problem of the Mathematical Gazette House", remarked Gh. Ţiţeica on this occasion.

In 1919 he graduated as an electrical engineer after graduating from the Ecole Supérieure de Electricité in Paris.

   In order to support the efforts of the Romanian delegation to the Paris Peace Conference (1919), of which Traian Lalescu was a member, the scientist wrote a monograph on the ethnographic problem of Banat, providing scientific arguments regarding this region belonging to Romania. Traian Lalescu was deputy of Caransebeş. He drafted and presented in Parliament a Report on the budget for the year 1925. He wrote philosophical dialogues on mathematical topics, being "primarily interested in the idea, the elegance of the proof, and the deep meanings of the theorems."

He campaigned for the establishment of the Polytechnic School of Timişoara, whose first rector (or director) was in 1920.

 Professor Traian Lalescu played an important role in the publication on March 15, 1921 of the Journal of Mathematics from Timişoara.

 Since 1990 he has been a post-mortem member of the Romanian Academy.

 From the work of Traian Lalescu we present:

ARTICLES AND BOOKS IN ROMANIAN:

1. Agenda bãncilor populare și metodul de coeficient Lalescu. București, 1906.

2. Introducere la teoria ecuațiunilor integrale. Bucuresti, 1911.

3. Dl. Spiru Haret ca om de știintã. În: Lui Spiru C. Haret, ale tale, dintru ale tale, la împlinirea celor șeasezeci ani. București, 1911.

4. Asupra variației valorilor caracteristice. București, Librãriile Socec și C. Sfetea; Viena, Gerold; Berlin, R. Friedlander und Sohn; Lipsca, O. Harrassowitz, 1912, Academia Românã.

5. Însumarea a doi sîmburi neortogonali. Notã: București, Librãriile Socec și C. Sfetea; Viena, Gerold; Berlin, R. Friedlaender und Sohn; Lipsca, O. Harrassowitz, 1913.

6. Raportul general asupra proiectului de buget al veniturilor și cheltuielilor Statului pe anul 1925, prezentat Adunãrii Deputaților. București, 1914.

7. Culegere de probleme de geometrie descriptivã și cosmografie (în colaborare cu Șt. N. Mirea). București, 1914.

8. Cuvântare la sãrbãtorirea ing. Constantin M. Mironescu. În vol.: Sãrbãtorirea domnului inginer inspector general Constantin M Mironescu, cu ocaziunea retragerii sale din funcţiunea de Director al școalei de Poduri și șosele. Lucrare întocmitã din initiativa Comitetului organizator de Dl. Prof. Traian Lalescu. București, Tipografia Profesionalã Dim. C. Ionescu, 1915.

9. Transcrierea dupã slove cirilice însoțitã de o notã biograficã și note explicative a cãrții Trigonometria de Gheorghe Lazãr. București, 1919. (Biblioteca Gazetei matematice).

10. Tratat de geometrie analiticã. Dreaptã, Plan, Conice, Cuadrice, Aplicatiile geometrice ale calculului infinitezimal. Editia întîi. București, 1920. (Biblioteca Gazetei matematice).

11. Tratat de geometrie analiticã. Ediția II. Fasc. I. București, 1923. (Biblioteca Gazetei Matematice).

12. Telefonia fãrã fir. București, Cartea Româneascã, 1923.

13. Calculul algebric. Polinoame, fracțiuni raționale. Biblioteca manualelor științifice, București ,1924.

14. Prefațã la cartea Dunãrea dintre Bazias și Turnu-Severin, Daniil Laitin. București, Tipografiile Române Unite, 1925. (Biblioteca Academiei București).

15. Curs de geometrie analiticã. Fascicula IV. Aplicațiile geometrice ale calculului infinitezimal. București, Tipografia F. Göbl și Fiii, 1927.(Biblioteca Gazetei Matematice)

16. Curs de geometrie analiticã. Dreaptã, plan, conice, cuadrice. București, 1931.(Biblioteca Universitarã).

17. Culegere de probleme de geometrie descriptivã. Ediția a doua. Revãzută de R. N. Racliș. București, 1935. (Publicațiunile Institutului Matematic Român).

18. Tratat de geometrie analiticã. Curs. Ediția 1938 revãzută. Caietul I—III. Caietul 1: Dreapta, planul; 2. Conicele; 3. Cuadricele. București, 1938.

19. Tratat de geometrie analiticã. Curs profesat la Politehnica din București de Traian Lalescu. Editia 1944, revãzută de Neculai Raclis. Cu o prefațã de D. Busilã. Caietul I. Dreapta, planul. București, Tipografia F. Göbl si Fiii, 1944.

20. Tratat de geometrie analiticã. Curs profesat. Ediţia 1938, revãzutã. Caietul 2, 3, ed. 1944. Bucuresti, F. Göbl si Fiii, 1938–1947.

21. Introducere la teoria ecuaţiilor integrale. București, Editura Academiei Republicii Populare Române, 1956.

22. Geometria triunghiului. Traducere îngrijitã de O. Sacter dupã ediţia a 2-a apãrutã în limba francezã în anul 1937. București, Editura tineretului, 1958.

23. Tratat de geometrie analiticã. Dreaptã, Plan, Conice, Cuadrice, Aplicaţiile geometrice ale calculului infinitezimal. Fasc. 3. Cuadrice. București, 1992. (Biblioteca Gazetei matematice).

24. Geometria triunghiului. Craiova, Editura Apollo, 1993.

SCIENTIFIC ARTICLES AND BOOKS IN A FOREIGN LANGUAGE:

1. Sur la composition des formes quadratiques. Extrait des Nouvelles Annales de Mathématiques, 4-e série, t. VII, Paris, avril, 1907.

2. Sur les solutions périodiques des equations différentielles linéaires. Paris, 1907.(CRASP).

3. Sur l'ordre de la fonction entière D (I) de Fredholm. In: Comptes Rendus des Séances de l'Académie des Sciences. Paris, 1907.(CRASP).

4. Sur le groupe des équations trinomes. Paris, 1907.(CRASP).

5. Sur une classe d'équations différentielles linéaires d'ordre infini. In: Comptes Rendus des Séances de l'Académie des Sciences. Paris, 1908.(CRASP).

6. Thèses présentées à la Faculté des Sciences de Paris. 1-ère Thèse sur l'équation de Volterra. 2-e Thèse. Propositions données par la faculté. Paris, 1908.

7. Sur l'équation de Volterra, 1-ère thèse. Propositions données par la Faculté, 2-e thèse. Soutenues [en] 1908, devant la commission d'examen. Thèses présentées à la Faculté des Sciences de Paris pour obtenir le grade de docteur en sciences mathématiques. Paris, Gauthier-Villars, 1908.

8. La théorie générale de Galois, Annales de la Faculté des Sciences de Toulouse, Paris, 1908.

9. Quelques remarques sur l'équation intégrale de Volterra. Bucarest,1909. (BSS).

10. Sur les solutions analytiques de l'équation In: Atti del IV Congresso internazionale dei matematici. Roma, 6–11 avrile 1908. Communicazione delle sezioni I e II. Vol. 2. Roma, 1909.

11. La théorie des équations intégrales linéaires d'ordre infini. Bucarest, 1910. (BSS).

12. Quelques remarques sur l'équation intégrale de Fredholm. Bucuresti, 1910. (BSS).

13. Sur l'équation de Lamé, nr. 1. Bucarest, 1910. (BSS).

14. Sur les noyaux résolvants. In: Comptes Rendus des Séances de l'Académie des Sciences. Paris, 1910.(CRASP).

15. Sur les noyaux symétriques gauches. In: Comptes Rendus des Séances de l'Académie des Sciences. Paris, 1910.(CRASP).

16. Théorèmes sur les valeurs caractéristiques. In: Comptes Rendus des Séances de l'Académie des Sciences. Paris, 1911. (CRASP).

17. Introduction à la théorie des équations intégrales. Avec une préface de M. Emile Picard. Paris, 1912. (CRASP).

18. Sur L'addition des noyaux non ortogonaux. Bucarest, 1913. (BSS).

19. Sur la notion des noyaux symétriques gauches définis. Sur une suite de noyaux remarquables. Sur une classe de noyaux brisés. Bucarest, 1915. (BSS).

20. 1. Sur un piège de la théorie des equations intégrales. 2. Un théorème sur les noyaux composés. Bucarest, 1915. (BSS).

21. Sur les solutions périodiques des equations différentielles du second ordre. Jassy, 1915.

22. Sur les problèmes bilocaux relatifs à l'équation différentielle linéaire du second ordre. Bucarest, 1916. (BSS).

23. Les classes de noyaux symétrisables. In: Comptes Rendus des Séances de l'Académie des Sciences. Paris, 1918. (CRASP).

24. Sur les séries trigonométriques et la théorie des équations intégrales. In: Comptes Rendus des Séances de l'Académie des Sciences. Paris, 1918. (CRASP).

25. Sur les fonctions polygonales périodiques. In: Comptes Rendus des Séances de l'Académie de Sciences. Paris, 1918. (CRASP).

26. Sur l'application des équations intégrales à la théorie des équations différentielles linéaires. In: Comptes Rendus des Séances de l'Académie des Sciences. Paris, 1918. (CRASP).

27. Les problèmes bilocaux pour l'équation différentielle linéaire du second ordre. Bucarest, 1918. (BSS).

28. Les équations différentielles linéaires d'ordre infini et l'équation de Fredholm. Roma, 1918.

29. Wstep do teorji równan calckowych. [Introducere în teoria ecuaþiilor integrale]. traducere din limba francezã de S. Mazurkiewicy. Warszawa, 1918.

30. Données statistiques sur le Banat. Paris, 1919.

31. Le problème éthnographique du Banat. Paris, 1919.

32. Sur l'approximation des fonctions par des séries trigonométriques. Bucarest, 1920. (BSS).

33. Sur la loi asymptotique de quelques classes de valeurs caractéristiques. Bucureşti, 1924. (BSS).

34. Sur un théorème de la théorie des noyaux simétrisables. Bucarest, Cultura Naţională, 1925. (Académie de Roumanie).

35. La géométrie du triangle. Deuxième édition. Avec une lettre de M. Émile Picard et une préface de M. Georges Tzitzeica. Bucarest, 1937.

36. La géométrie du triangle. Paris, Librairie Vuibert, 1952.

ARTICLES PUBLISHED IN MATHEMATICAL GAZETTE:

1. Asupra însumării de factoriale consecutive, anul V, 1899-1900, pp. 277-281.

2. Câteva relaţiuni trigonometrice, anul VI, 1900-1901, pp. 197-200.

3. Asupra unei chestiuni de analizăcombinatorie, anul VII, 1901-1902.

4. O generalizare, anul VIII, 1902-1903, pp. 197.

5. Proprietăţi ale cercului ortocentroidal, anul IX, 1903-1904, pp. 31-34.

6. Asupra unei integrale duble, anul X, 1904-1905, pp. 227-229.

7. Asupra substituţiilor circulare, anul XI, 1905-1906, pp. 270-273.

8. Un exemplu de aproximaţii successive, anul XIII, 1907-1908, pp. 97-102.

9. Asupra unei formule a lui Riemann, anul XIV, 1908-1909, pp. 33-35.

10. O problemă de algebră, anul XIV, 1908-1909, pp. 68-72.

11. Asupra unei formule a lui Riemann-Hadamard, anul XIV, 1908-1909, pp. 99-103.

12. Criterii pentru recunoaşterea cuadricelor, anul XIV, 1908-1909, pp. 232-234.

13. Careacterizarea conicelor date prin 5 puncte, anul XV, 1909-1910, pp. 193-194.

14. Perpendiculara comună la două drepte, anul XVI, 1910-1911, pp. 84-86.

15. Asupra pendulului lui Foucault, anul XVI, 1910-1911, pp. 404-406.

16. Privire istorică asupra teoriei numerelor, anul XVIII, 1912-1913, pp. 85-91. (Acest articol a fost tradus în limba spaniolă de Bernard Baidaff şi tipărită în revista argentiniană ''Boletin matematico''. Lucrarea a apărut la Buenos Aires, sub titlul Una mirade historica de la teoria de los numerous, vol. XIII, pp. 76-78 şi 105-111, 1940).

17. Perspectiva în studiul geometriei descriptive, anul XVIII, 1912-1913, pp. 439-443.

18. Nicolae Culianu, anul XXI, 1915-1916, pp. 161-166.

19. Asupra unui punct remarcabil al triunghiului, anul XXI, 1915-1916, pp. 241-243.

20. Viaţa şi activitatea lui Gheorghe Lazăr, anul XXII, 1916-1917, pp. 151-156, 177-185, 207-209 şi 217-221.

21. Bibliografia matematică românească, anul XXII, 1916-1917, pp. 270-271.

22. Cărţi de matematici din Transilvania, anul XXII, 1916-1917, pp. 300-306.

23. Cărţi şi manuscrise greceşti de matematică din ţările române, anul XXIII, 1917-1918, pp. 107-110, 130-132.

24. Catalogul cărţilor şi manuscriselor româneşti de matematică la expoziţia de la Iaşi din 1885, anul XXIII, 1917-1918, p. 178.

25. Câtul a două polinoame, anul XXVII, 1921-1922, pp. 105-111.

26. Asupra unei colineaţiuni a conicelor, anul XXVII, 1921-1922, pp. 272-275.

27. Unul din primii profesori de matematici: Simion Marcovici, anul XXIX, 1923-1924, pp.41-43.

28. Câteva date asupra lui Simion Marcu zis Marcovici, ca profesor de matematică, anul XXIX, 1923-1924, pp. 121-123.

ARTICLES PUBLISHED IN THE MATHEMATICAL JOURNAL OF TIMISOARA:

1. Rezolvarea ineglităţilor algebrice, anul I, 1921, pp. 3-7.

2. Probleme de Algebră superioară, anul I, nr. 2, 15 aprilie 1921, pp. 21-24.

3. Construcţii geometrice, anul I, nr. 5, iulie 1921, pp. 67-69.

4. Discuţia construcţiunilor geometrice, anul I, nr. 6, august 1921, pp. 83-85.

5. Dreapta lui Simson, anul II, nr. 10, decembrie 1922.

6. Serii convergente şi serii divergente, anul III, 1923, pp. 35-37.
7. Simetrie şi omogeneitate, anul III, nr. 6, 1923, pp. 83-85.
8. Patrulatere remarcabile, anul III, nr. 12, februarie 1924, pp. 179-180.
9. Diviziunea polinoamelor, anul V, nr. 1, martie 1926, pp. 4-6.

STATUES:

Bust of Traian Lalescu from the "Polytechnic" University of Timișoara - sculptor Corneliu Medrea
Bust of Traian Lalescu from the "Polytechnic" University of Timișoara - sculptor Peter Jecza
Bust of Traian Lalescu from the University of Bucharest - Faculty of Mathematics and Informatics - sculptor Peter Jecza

STREETS:

Traian Lalescu Street from Timișoara
Traian Lalescu Street from Reșița
Traian Lalescu Street from Oradea
Traian Lalescu Street from Craiova

HIGH SCHOOLS:

„Traian Lalescu" National College of Informatics - Hunedoara, Hunedoara County
„Traian Lalescu" National College - Reșița, Caraș-Severin County „Traian Lalescu" Theoretical High School - Orșova - Mehedinți County
„Traian Lalescu" Theoretical High School - Mehadia, Caraș-Severin County
"Traian Lalescu" Theoretical High School - Branesti, Ilfov County "Traian Lalescu" High School - Bucharest - (private high school, established in 1992, accredited in 2009)

OTHER:

The documentary film "Traian Lalescu - the right to memory", 48 min - TVR Cultural, 2008.
Anniversary Medal "Traian Lalescu - 125 years since birth"
"Traian Lalescu" presentation panel from the Faculty of Mathematics and Informatics of the University of Bucharest
Short film about Traian Lalescu – 70's years- TVR
OSTL-"Traian Lalescu" Student Association-"Politehnica" University of Timișoara- Faculty of Constructions and Department of Communication and Foreign Languages, established in 2007.
www.ostl.ro
https://www.youtube.com/watch?v=tNJfRYKb8DQ
https://www.youtube.com/watch?v=8IYbRAPAddw

EVOCATIONS:

Emile Picard: "Lalescu's very lively intelligence allowed him to immediately reach the heart of a problem; that is why his texts have that spontaneity that makes them particularly attractive. His curious spirit was interested in the most varied fields of mathematics, and we often walked together through the Luxembourg Garden, discussing various subjects of philosophy of science".

Grigore Moisil: "The prodigious activity of this great scientist (…) is for us an invaluable scientific legacy".

Edmond Nicolau: "The history of mathematics in our country places professor Traian Lalescu together with Gheorghe Ţiţeica and Dimitrie Pompei in the group of founders of the Romanian mathematics school".

Ion Ionescu-Bizeţ: "Lalescu's appearance in the world was like a comet that shone and shone wonderfully and at the same time amazed and scared with its unusually long tail".

Gheorghe Ţiţeica: "Lalescu's head was worth much more than ten estates".

The presented ones characterize the complex personality, encyclopedic spirit and the erudition of Traian Lalescu. In this sense, the finding of the academician Solomon Marcus is convincing, who classifies mathematicians in two classes: those of the ant type, who insist in a certain direction throughout their lives and those of the bee type, who do not remain in the same place, but ''flies from flower to flower''. Solomon Marcus, places Lalescu in the class of bee researchers:

''Albina Lalescu was not satisfied with the flowers offered by mathematics, but ventured to the flowers of Romanian history, finance, sociology, physics, engineering, linguistics, history of mathematics textbooks, history of mathematics, propagation in masses of scientific culture, philosophy, etc''.

REFERENCES ABOUT THE LIMIT OF TRAIAN LALESCU

1. **D.M. Bătineţu-Giurgiu**, *Şiruri,* Editura Albatros, Bucureşti, 1979.

2. **D.M. Bătineţu-Giurgiu**, *Şiruri Lalescu*, R.M.T., anul XX (1989), pp. 37-38.

3. **D.M. Bătineţu-Giurgiu**, *Şirurile Lalescu şi funcţia Euler de speţa a doua. Funcţii Euler-Lalescu*, Gazeta Matematică seria A, nr.1 /1990, pp. 21-26.

4. **D.M. Bătineţu-Giurgiu**, *Asupra unei generalizări a şirului lui Traian Lalescu. Metode de abordare,* Gazeta Matematică, nr. 8-9/1990, pp. 219-224.

5. **D.M. Bătineţu-Giurgiu, N. Stanciu** – *120 Years of Lalescu Sequences*, Romanian Mathematical Magazine, February 2020.

6. **D.M. Bătineţu-Giurgiu, D. Sitaru, N. Stanciu -** *New classes of sequences-functions and famous limits*– Octogon Mathematical Magazine, Vol. 27, No.2, October, 2019, 784-797.

7. **D.M. Bătineţu-Giurgiu, D. Sitaru, N. Stanciu -***Two classes of Lalescu's sequences* – Octogon Mathematical Magazine, Vol. 27, No. 2, October, 2019, 805-813.

8. **D.M. Bătineţu-Giurgiu, M. Bencze, D. Sitaru, N. Stanciu -** *Some limits of Traian Lalescu type with Fibonacci and Lucas numbers (II)* – Octogon Mathematical Magazine, Vol. 27, No. 1, April, 2019, 101-111.

9. **D.M. Bătineţu-Giurgiu, M. Bencze, D. Sitaru, N. Stanciu -** *Some limits of Traian Lalescu type (II)*– Octogon Mathematical Magazine, Vol. 27, No. 1, April, 2019, 184-209.

10. **D.M. Bătineţu-Giurgiu, M. Bencze, D. Sitaru, N. Stanciu -** *Some limits of definite integrals of Lalescu's type* – Octogon Mathematical Magazine, Vol. 27, No. 1, April, 2019, 312-328.

11. **D.M. Bătineţu-Giurgiu, M. Bencze, D. Sitaru, N. Stanciu -** *Some limits of Traian Lalescu type (II)*– Octogon Mathematical Magazine, Vol. 26, No. 2, October, 2018, 664-690.

12. **D.M. Bătineţu-Giurgiu, M. Bencze, N. Stanciu -** *Some limits of Traian Lalescu type with Fibonacci and Lucas numbers* – Octogon Mathematical Magazine, Vol. 26, No. 1, April, 2018, 54-87.

13. **D.M. Bătineţu-Giurgiu, M. Bencze, N. Stanciu -** *Some limits of Traian Lalescu type (I)* – Octogon Mathematical Magazine, Vol. 26, No. 1, April, 2018, 116-136.

14. **D.M. Bătineţu-Giurgiu, M. Bencze, D. Sitaru, N. Stanciu -***Certain classes of Lalescu sequence*– Octogon Mathematical Magazine, Vol. 26, No. 1, April, 2018, 243-251.

15. **D.M. Bătineţu-Giurgiu, N. Stanciu -***Several results of some classes of sequences*– The Pentagon – Volume 73, Number 2, Spring 2014, 10-24.

16. **D.M. Bătineţu-Giurgiu, A. Kotronis, N. Stanciu -***Calculating the limits of some real sequences*– Math Problems, Issue 1, 2014, 252-257.

17. **D.M. Bătineţu-Giurgiu, N. Stanciu -** *New methods for calculations of some limits*– The Teaching of Mathematics, Vol. XVI, Nr. 2, 2013, 82-88.

18. **D.M. Bătineţu-Giurgiu, N. Stanciu -** *120 de ani de la limita şirului lui Traian Lalescu* – Sclipirea Minţii, Nr. 25, Mai, 2020, 6-7.

19. **D.M. Bătineţu-Giurgiu, N. Stanciu, José Luis Díaz-Barrero -** *The Last Three Decades of Lalescu Limit* – Arhimede Mathematical Journal, Vol. 7, No.1, 2020, 18-28.

20. D.M. Bătineţu-Giurgiu, N. Stanciu − *120 de ani de la limita lui Traian Lalescu − Recreaţii Matematice*, Nr. 2, Iulie-Decembrie, 2020, 104-106.

21. N. Stanciu - *Limits of Lalescu Type* − Journal of Mathematics and Statistics Research, Research Article,Volume 2, Issue 1, 2020, 1-16.

22. D. Sitaru, N. Stanciu − *Traian Lalescu limit - 120 years, D.M. Bătineţu-Giurgiu − 85 years*, Romanian Mathematical Magazine, July 20, 2020.

Proposed problem for RMM

Proposed by D.M. Bătineţu-Giurgiu, Neculai Stanciu − Romania

Let the positive real sequence $(a_n)_{n\geq 1}$, such that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n \sqrt[n]{n!}} = a \in \mathbb{R}_+^*$. Compute:

$$\lim_{n\to\infty}\left(\sqrt[n+1]{a_{n+1}} - \sqrt[n]{a_n}\right)$$

Solution:

$$\lim_{n\to\infty} \frac{\sqrt[n]{a_n}}{n} = \lim_{n\to\infty} \sqrt[n]{\frac{a_n}{n^n}} \overset{C-D'A}{=} \lim_{n\to\infty} \frac{a_{n+1}}{(n+1)^n}\cdot\frac{n^n}{a_n} = \lim_{n\to\infty} \frac{a_{n+1}}{a_n\sqrt[n]{n!}}\cdot\left(\frac{n}{n+1}\right)^{n+1}\cdot\frac{\sqrt[n]{n!}}{n!} =$$

$$= \lim_{n\to\infty} \frac{a}{e}\sqrt[n]{\frac{n!}{n^n}} \overset{C-D'A}{=} \frac{a}{e}\lim_{n\to\infty}\frac{(n+1)!}{(n+1)^{n+1}}\cdot\frac{n^n}{n!} = \frac{a}{e}\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = \frac{a}{e^2}$$

$$\sqrt[n+1]{a_{n+1}} - \sqrt[n]{a_n} = \sqrt[n]{a_n}\cdot(u_n - 1) = \frac{\sqrt[n]{a_n}}{n}\cdot\frac{u_n-1}{\ln u_n}\ln u_n^n, \text{ where}$$

$$u_n = \frac{\sqrt[n+1]{a_{n+1}}}{\sqrt[n]{a_n}} = \frac{\sqrt[n+1]{a_{n+1}}}{n+1}\cdot\frac{n+1}{n}\cdot\frac{n}{\sqrt[n]{a_n}}, \forall n\geq 2$$

$$\lim_{n\to\infty} u_n = 1, \lim_{n\to\infty}\frac{u_n-1}{\ln u_n} = 1, \lim_{n\to\infty} u_n^n = \lim_{n\to\infty}\frac{a_{n+1}}{a_n}\cdot\frac{1}{\sqrt[n+1]{a_{n+1}}} =$$

$$= \lim_{n\to\infty}\frac{a_{n+1}}{a_n\sqrt[n]{n!}}\cdot\frac{n+1}{\sqrt[n+1]{a_{n+1}}}\cdot\frac{n}{n+1}\cdot\frac{\sqrt[n]{n!}}{n} = a\cdot\frac{e^2}{a}\cdot 1\cdot\frac{1}{e} = e$$

Hence, $\lim_{n\to\infty}\left(\sqrt[n+1]{a_{n+1}} - \sqrt[n]{a_n}\right) = \frac{a}{e^2}\cdot 1\cdot\ln e = \frac{a}{e^2}$