A TRIPLE GENERALIZATION

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Abstract: *In this short math-note we will give a triple generalization for famous Ionescu-Weitzenböck, Gordon's and Goldner's inequalities.*

Main Result:

If $x, y, z \geq 0$ then in any triangle ABC holds:

$$a^{x+1}b^y c^z + a^y b^z c^{x+1} + a^z b^{x+1} c^y \geq 2^{x+y+z} \cdot (\sqrt[4]{3})^{3-x-y-z} \cdot (\sqrt{F})^{x+y+z+1}$$

Proof. We have:

$$a^{x+1}b^y c^z + a^y b^z c^{x+1} + a^z b^{x+1} c^y \overset{AM-GM}{\geq} 3 \cdot \sqrt[3]{a^{x+1}b^y c^z \cdot a^y b^z c^{x+1} \cdot a^z b^{x+1} c^y} =$$

$$= 3 \cdot \sqrt[3]{(abc)^{x+y+z+1}} = 3\left(\sqrt[3]{(abc)^2}\right)^{\frac{x+y+z+1}{2}} =$$

$$= 3 \cdot 3^{\frac{x+y+z+1}{2}} \cdot \left(\sqrt[3]{(abc)^2}\right)^{\frac{x+y+z+1}{2}} \cdot 3^{-\frac{x+y+z+1}{2}} =$$

$$= 3^{3-\frac{x+y+z+1}{2}} \cdot \left(3\sqrt[3]{(abc)^2}\right)^{\frac{x+y+z+1}{2}} \overset{Carlitz}{\geq}$$

$$= 3^{1-\frac{x+y+z+1}{2}} \cdot (4\sqrt{F})^{\frac{x+y+z+1}{2}} =$$

$$= 3^{1-\frac{x+y+z+1}{2}} \cdot 4^{\frac{x+y+z+1}{2}} \cdot (\sqrt{3})^{\frac{x+y+z+1}{2}} \cdot F^{\frac{x+y+z+1}{2}} =$$

$$= 3^{1-\frac{x+y+z+1}{2}+\frac{x+y+z+1}{4}} \cdot 2^{x+y+z+1} \cdot (\sqrt{F})^{x+y+z+1} =$$

$$2^{x+y+z+1} \cdot 3^{\frac{3-x-y-z}{4}} \cdot (\sqrt{F})^{x+y+z+1}$$

$$= 2^{x+y+z+1} \cdot (\sqrt{4})^{3-x-y-z} \cdot (\sqrt{F})^{x+y+z+1}.$$

Observation 1. If $x = 1$, $y = 0$, $z = 0$, then:

$$a^{1+1} \cdot b^0 \cdot c^0 + a^0 \cdot b^0 \cdot c^{1+1} + a^0 \cdot b^{1+1} \cdot c^0 \geq 2^{1+0+0+1} \cdot (\sqrt[4]{3})^{3-1-0-0} \cdot (\sqrt{F})^{1+0+0+1} = 4\sqrt{3}F$$

Observation 2. If $x = z = 0$, $y = 1$, then:

$$a^{0+1} \cdot b^1 \cdot c^0 + a^1 \cdot c^0 \cdot c^{0+1} + a^0 \cdot b^{0+1} \cdot c^1 \geq 2^{0+1+0+1} \cdot (\sqrt[4]{3})^{3-0-1-0}(\sqrt{F})^{0+1+0+1}$$

$$ab + bc + ca \geq 4\sqrt{3}F; \ (Gordon)$$

Observation 3. If $x = 3$, $y = z = 0$, then:

$$a^{3+1} \cdot b^0 \cdot c^0 + a^0 \cdot b^0 \cdot c^{3+1} + a^0 \cdot b^{3+1} \cdot c^0 \geq 2^{3+0+1+1} \cdot (\sqrt[4]{3})^{3-3-0-0} \cdot (\sqrt{F})^{3+0+0+1}$$

$$a^4 + b^4 + c^4 \geq 16F^2; \ (Goldner)$$

REFERENCES

[1]. **ROMANIAN MATHEMATICAL MAGAZINE**-www.ssmrmh.ro

[2]. **OCTOGON MATHEMATICAL MAGAZINE**