Proposed by: Prof. Dan Sitaru

$$\Omega = \lim_{n\to\infty} \sqrt[n]{\sum_{k=1}^{n} \binom{2k}{k}\binom{2n-2k}{n-k}\frac{1}{(k+1)(n-k+1)}}$$

Solution Proposed by : Surjeet Singh , India

We know that $\displaystyle\sum_{n=0}^{\infty}\binom{2n}{n}x^n = \frac{1}{\sqrt{1-4x}}$ for $|x| < 4^{-1}$ by integrating this we get

$$\sum_{k=0}^{\infty}\binom{2k}{k}\frac{x^{k+1}}{k+1} = \frac{1}{2}\left(1-\sqrt{1-4x}\,\right) \text{ *Using cauchy's Product*}$$

$$\left(\sum_{k=0}^{\infty}\binom{2k}{k}\frac{x^{k+1}}{k+1}\right)^2 = \sum_{n=0}^{\infty}c_n x^{n+2} = \frac{1}{2}\left(-2x+1-\sqrt{1-4x}\right)$$

Where $\displaystyle c_n = \sum_{k=0}^{n}\binom{2k}{k}\binom{2n-2k}{n-k}\frac{1}{(k+1)(n-k+1)}$

Compare cofficients $\displaystyle\sum_{n=0}^{\infty}c_n x^{n+2} = -x + \sum_{k=0}^{\infty}\binom{2k}{k}\frac{1}{k+1}x^{k+1}$

Hence $\displaystyle c_n = \binom{2n+2}{n+1}\frac{1}{n+2}$ Now we will find our Ω

$$\sum_{k=1}^{n}\binom{2k}{k}\binom{2n-2k}{n-k}\frac{1}{(k+1)(n-k+1)} = \frac{1}{n+2}\binom{2n+2}{n+1} - \frac{1}{n+1}\binom{2n}{n}$$

$$= \frac{3n}{(n+1)(n+2)}\binom{2n}{n} \text{ Hence } \Omega = 4\lim_{n\to\infty}\sqrt[n]{\frac{3n}{(n+1)(n+2)}} = 4$$