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ROMANIAN MATHEMATICAL MAGAZINE
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PROBLEMS WITH NATURAL NUMBERS

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Abstract: *In this paper I will present a set of problems with dividing a natural number from a given format to another given natural number, for which we want to find out and the rest of the quotient and the remainder is relatively similar, differing by computational devices in the case of division into numbers of different forms.*

Application 1. ***Find out how the remainder of the division of the number $\mathbf{10^{3k}}, \boldsymbol{k} \in \mathbb{N}^*$ to the number*** $\underbrace{\overline{\mathbf{3\ 003\ 003 \ldots 003}}}_{\mathbf{3k-2-digits}}$.

Solution. We have: $\underbrace{\overline{9999999\ldots9}}_{3k-digits} : 111 = \underbrace{\overline{9009009\ldots009}}_{3k-2-digits} \Rightarrow$

$$\underbrace{\overline{9999999\ldots9}}_{3k-digits} = 111 \cdot \underbrace{\overline{9009009\ldots009}}_{3k-2-digits} \Rightarrow 10^{3k} - 1 = 111 \cdot \underbrace{\overline{9009009\ldots009}}_{3k-2-digits}$$

$$\Rightarrow 10^{3k} = 111 \cdot \underbrace{\overline{9009009\ldots009}}_{3k-2-digits} + 1 \Rightarrow 10^{3k} = 111 \cdot 3 \cdot \underbrace{\overline{3003003\ldots003}}_{3k-2-digits} + 1$$

$$= 333 \cdot \underbrace{\overline{3003003\ldots003}}_{3k-2-digits} + 1$$

So, we have $\underbrace{\overline{3003003\ldots003}}_{3k-2-digits} < 10^{3k}$, hence the extend of the division is 333 and the rest of the division is 1.

Application 2. ***Find out the remainder of the division of the number $\mathbf{10^{3k}}, \boldsymbol{k} \in \mathbb{N}^*$ to the number*** $\underbrace{\overline{\mathbf{8\ 008\ 008 \ldots 008}}}_{\mathbf{3k-2-digits}}$.

Solution. We can write:

$$10^{3k} - 1 = 111 \cdot \underbrace{\overline{9009009\ldots009}}_{3k-2-digits} \Rightarrow 10^{3k} = 111 \cdot \underbrace{\overline{9009009\ldots009}}_{3k-2-digits} + 1$$

$$10^{3k} = 999 \cdot \underbrace{\overline{1001001\ldots001}}_{3k-2-digits} + 1 = (124 \cdot 8 + 7) \cdot \underbrace{\overline{1001001\ldots001}}_{3k-2-digits} + 1$$

$$10^{3k} = 124\cdot 8\cdot \underbrace{\overline{1001001\ldots001}}_{3k-2-digits} + 7\cdot \underbrace{\overline{1001001\ldots001}}_{3k-2-digits} + 1$$

$$10^{3k} = 124\cdot \underbrace{\overline{8008008\ldots008}}_{3k-2-digits} + \underbrace{\overline{7007007\ldots008}}_{3k-2-digits}$$

How $\underbrace{\overline{8008008\ldots008}}_{3k-2-digits} > \underbrace{\overline{7007007\ldots008}}_{3k-2-digits}$, hence the extend of the division is 124 and the rest of the division is $\underbrace{\overline{7007007\ldots008}}_{3k-2-digits}$.

Application 3. ***Find out the remainder of the division of the number 10^{3k}, $k\in\mathbb{N}^*$ to the number $\underbrace{\overline{19\,019\,019\ldots019}}_{3k-1-digits}$.***

Solution. We can write:

$$10^{3k} = 999\cdot \underbrace{\overline{1001\ldots001}}_{3k-2-digits} + 1 = (52\cdot 19 + 11)\cdot \underbrace{\overline{1001001\ldots001}}_{3k-2-digits} + 1$$

$$10^{3k} = 52\cdot 19\cdot \underbrace{\overline{1001001\ldots001}}_{3k-2-digits} + 11\cdot \underbrace{\overline{1001\ldots001}}_{3k-2-digits}$$

But $\underbrace{\overline{19019019\ldots019}}_{3k-1-digits} = 19\cdot \underbrace{\overline{1001001\ldots001}}_{3k-2-digits}$ and $\underbrace{\overline{11011011\ldots011}}_{3k-1-digits} = 11\cdot \underbrace{\overline{1001001\ldots001}}_{3k-2-digits}$, then we get:

$$10^{3k} = 52\cdot \underbrace{\overline{19019019\ldots019}}_{3k-1-digits} + \underbrace{\overline{11011011\ldots011}}_{3k-1-digits} + 1 =$$

$$= 52\cdot \underbrace{\overline{19019019\ldots019}}_{3k-1-digits} + \underbrace{\overline{11011011\ldots012}}_{3k-1-digits}.$$

Application 4. ***Find out the remainder of the division of the number 10^{4k}, $k\in\mathbb{N}^*$ to the number $\underbrace{\overline{2022\,2022\ldots2022}}_{4k-digits}$.***

Solution. We can write:

$$\underbrace{\overline{9999\ldots99}}_{4k-digits} = 9999\cdot \underbrace{\overline{10001\ldots0001}}_{4k-3-digits}$$

$$10^{4k} = 9999\cdot \underbrace{\overline{10001\ldots0001}}_{4k-3-digits} + 1 = (4\cdot 2022 + 1911)\cdot \underbrace{\overline{10001\ldots0001}}_{4k-3-digits} + 1$$

$$10^{4k} = 4\cdot 2022\cdot \underbrace{\overline{10001\ldots0001}}_{4k-3-digits} + 1911\cdot \underbrace{\overline{10001\ldots0001}}_{4k-3-digits} + 1$$

But $\underbrace{\overline{2022\,2022\ldots 2022}}_{4k-digits} = 2022\cdot\underbrace{\overline{10001\ldots 0001}}_{4k-3-digits}$ and $\underbrace{\overline{19111911\ldots 1911}}_{4k-digits} =$

$= 1911\cdot\underbrace{\overline{10001\ldots 0001}}_{4k-3-digits}$, then:

$$10^{4k} = 4\cdot\underbrace{\overline{2022\,2022\ldots 2022}}_{4k-digits} + \underbrace{\overline{19111911\ldots 1911}}_{4k-digits} + 1$$

$$10^{4k} = 4\cdot\underbrace{\overline{2022\,2022\ldots 2022}}_{4k-digits} + \underbrace{\overline{19111911\ldots 1912}}_{4k-digits}$$

But $\underbrace{\overline{19111911\ldots 1912}}_{4k-digits} < \underbrace{\overline{2022\,2022\ldots 2022}}_{4k-digits}$, hence the extend of the division is

$\underbrace{\overline{2022\,2022\ldots 2022}}_{4k-digits}$ and the rest of the division is $\underbrace{\overline{19111911\ldots 1912}}_{4k-digits}$.

Application 5. ***Find out the remainder of the division of the number 10^{3k}, $k\in\mathbb{N}^*$ to the number 37.***

Solution. We have $37\cdot 3 = 111$ and $\underbrace{\overline{9999999\ldots 9}}_{3k-digits} : 111 = \underbrace{\overline{9009009\ldots 009}}_{3k-2-digits} \Rightarrow$

$$\underbrace{\overline{9999999\ldots 9}}_{3k-digits} = 37\cdot 3\cdot\underbrace{\overline{9009009\ldots 009}}_{3k-2-digits}$$

$$10^{3k} - 1 = 37\cdot 3\cdot\underbrace{\overline{9009009\ldots 009}}_{3k-2-digits} \Rightarrow 10^{3k} = 37\cdot\underbrace{\overline{27027027\ldots 027}}_{3k-1-digits} + 1$$

We have: $\underbrace{\overline{27027027\ldots 027}}_{3k-1-digits} < 10^{3k}$, hence the extend of the division is $\underbrace{\overline{27027027\ldots 027}}_{3k-1-digits}$

and the rest is 1.

Application 6. ***Find out the remainder of the division of the number $\underbrace{\overline{12333\ldots 33320}}_{3k+2-digits}$, $k\in\mathbb{N}^*$ to the number $\underbrace{\overline{1001\ldots 001\,001}}_{3k-2-digits}$.***

Solution. We have: $\underbrace{\overline{12333\ldots 33320}}_{3k+2-digits} = \underbrace{\overline{12333\ldots 33321}}_{3k+2-digits} - 1 = 3\cdot 37\cdot\underbrace{\overline{111\ldots 1111}}_{3k-digits} - 1 =$

$$= 111\cdot 111\cdot\underbrace{\overline{1001\ldots 001001}}_{3k-2} - 1 = 12321\cdot\underbrace{\overline{1001\ldots 001001}}_{3k-2} - 1 =$$

$$= 12320\cdot\underbrace{\overline{1001\ldots 001001}}_{3k-2} + \underbrace{\overline{1001\ldots 001001}}_{3k-2} - 1 =$$

$$= 12320 \cdot \underbrace{\overline{1001 \dots 001001}}_{3k-2} + \underbrace{\overline{1001 \dots 001000}}_{3k-2}$$

So, the extend of the division is 12320 and the rest is $\underbrace{\overline{1001 \dots 001001}}_{3k-2}$.

Application 7. ***Find out the remainder of the division of the number***

$$\underbrace{\overline{\mathbf{3699 \dots 999\,963}}}_{\mathbf{3k+2-digits}},\ \boldsymbol{k} \in \mathbb{N}^* \textbf{\textit{ to the number }} \underbrace{\overline{\mathbf{3003 \dots 003\,003}}}_{\mathbf{3k-2-digits}}.$$

Solution. We have:

$$\underbrace{\overline{3699 \dots 999\,963}}_{3k+2-digits} : 9 : 37 : 37 = \underbrace{\overline{4111 \dots 111107}}_{3k+1} : 37 : 37 = \underbrace{\overline{111 \dots 1111}}_{3k-digits} : 37$$

$$\underbrace{\overline{3699 \dots 999\,963}}_{3k+2-digits} : 9 : 37 : 37 = \underbrace{\overline{111 \dots 1111}}_{3k-digits} : 37 = 111 \cdot \underbrace{\overline{1001 \dots 001001}}_{3k-2} : 37$$

$$\underbrace{\overline{3699 \dots 999\,963}}_{3k+2-digits} : 9 : 37 : 37 = 3 \cdot \underbrace{\overline{1001 \dots 001001}}_{3k-2} = \underbrace{\overline{3003 \dots 003003}}_{3k-2}$$

So, $\underbrace{\overline{3699 \dots 999\,963}}_{3k+2-digits} = 3 \cdot 37 \cdot 37 \cdot \underbrace{\overline{3003 \dots 003003}}_{3k-2} = 4107 \cdot \underbrace{\overline{3003 \dots 003003}}_{3k-2}$, hence the extend of the division is 4107 and the rest is 0.

Or, $\underbrace{\overline{3699 \dots 999\,963}}_{3k+2-digits} + 37 = \underbrace{\overline{370000 \dots 000}}_{3k+2-digits} = 37 \cdot 10^{3k}$, hence we get:

$$\underbrace{\overline{3699 \dots 999\,963}}_{3k+2-digits} + 37 = 37\left(\underbrace{\overline{999999 \dots 999}}_{3k-digits} + 1\right)$$

$$\underbrace{\overline{3699 \dots 999\,963}}_{3k+2-digits} = 37 \cdot \underbrace{\overline{999999 \dots 999}}_{3k-digits}$$

$$\underbrace{\overline{3699 \dots 999\,963}}_{3k+2-digits} = 37 \cdot 9 \cdot \underbrace{\overline{111111 \dots 111}}_{3k-digits}$$

$$\underbrace{\overline{3699 \dots 999\,963}}_{3k+2-digits} = 37 \cdot 9 \cdot 111 \cdot \underbrace{\overline{1001 \dots 001}}_{3k-2-digits}$$

$$\underbrace{\overline{3699 \dots 999\,963}}_{3k+2-digits} = 37 \cdot 3 \cdot 27 \cdot 3 \cdot \underbrace{\overline{1001 \dots 001}}_{3k-2-digits}$$

$\underbrace{\overline{3699 \dots 999\,963}}_{3k+2-digits} = 37 \cdot 3 \cdot 37 \cdot \underbrace{\overline{3003 \dots 003}}_{3k-2-digits}$,hence the extend of the division is 4107 and the rest is 0.

Application 8. ***Find out the remainder of the division of the number***

$$\underbrace{\overline{\mathbf{27\,999 \dots 999\,974}}}_{\mathbf{3k+2-digits}},\ \boldsymbol{k} \in \mathbb{N}^* \textbf{\textit{ to the number }} \underbrace{\overline{\mathbf{37\,037 \dots 037}}}_{\mathbf{3k-1-digits}}.$$

Solution. We have: $\underbrace{\overline{27\,999\ldots999\,972}}_{3k+2-digits}+28=\underbrace{\overline{28\,000\ldots000\,000}}_{3k+2-digits}=28\cdot10^{3k}$

$$\underbrace{\overline{27\,999\ldots999\,972}}_{3k+2-digits}+28=28\left(\underbrace{\overline{999999\ldots999}}_{3k-digits}+1\right)$$

$$\underbrace{\overline{27\,999\ldots999\,972}}_{3k+2-digits}=28\cdot\underbrace{\overline{999999\ldots999}}_{3k-digits}=28\cdot27\cdot37\cdot\underbrace{\overline{1001\ldots001}}_{3k-2-digits}=$$

$=756\cdot\underbrace{\overline{37\,037\ldots037}}_{3k-1-digits}$, hence the extend of the division is 756 and the rest is 2.

Application 9. ***Find out the remainder of the division of the number 10^{3k}, $k\in\mathbb{N}^*$ to the number$\underbrace{\overline{18\,018\,018\ldots018}}_{3k-1-digits}$.***

Solution. We have: $10^{3k}=999\cdot\underbrace{\overline{1\,001\ldots001}}_{3k-2-digits}+1=(54\cdot18+27)\underbrace{\overline{1\,001\ldots001}}_{3k-2-digits}+1$

$$10^{3k}=54\cdot18\cdot\underbrace{\overline{1\,001\ldots001}}_{3k-2-digits}+27\cdot\underbrace{\overline{1\,001\ldots001}}_{3k-2-digits}$$

But $\underbrace{\overline{18\,018\,018\ldots018}}_{3k-1-digits}=18\cdot\underbrace{\overline{1\,001\ldots001}}_{3k-2-digits}$ and $\underbrace{\overline{27027027\ldots027}}_{3k-1-digits}=27\cdot\underbrace{\overline{1\,001\ldots001}}_{3k-2-digits}$. So,

$$10^{3k}=54\cdot\underbrace{\overline{18\,018\,018\ldots018}}_{3k-1-digits}+\underbrace{\overline{27027027\ldots027}}_{3k-1-digits}+1$$

$$10^{3k}=54\cdot\underbrace{\overline{18\,018\,018\ldots018}}_{3k-1-digits}+\underbrace{\overline{18\,018\,018\ldots018}}_{3k-1-digits}+\underbrace{\overline{9\,009\,009\ldots009}}_{3k-2}+1$$

$$10^{3k}=55\cdot\underbrace{\overline{18\,018\,018\ldots018}}_{3k-1-digits}+\underbrace{\overline{9\,009\,009\ldots010}}_{3k-2}.$$

Application 10. ***Find out the remainder of the division of the number 10^{nk}, $k\in\mathbb{N}^*$ to the number$\underbrace{\overline{2\underbrace{0\ldots0}_{n-1\,digits}2\underbrace{0\ldots0}_{n-1-digits}2\underbrace{0\ldots0}_{n-1-digits}2}}_{(k-1)n+1-cifre}$.***

Solution. Let be $\underbrace{\overline{9999\ldots999}}_{nk-digits}:\underbrace{\overline{111\ldots11}}_{n-digits}=\underbrace{\overline{9\underbrace{\underbrace{0\ldots0}_{n-1\,digits}9\underbrace{0\ldots0}_{n-1\,digits}9\underbrace{0\ldots0}_{n-1\,digits}9}_{(k-1)-terms\ of\ \underbrace{\overline{0\ldots09}}_{n-\,digits}}}}_{(k-1)n+1-digits}$

$$10^{nk}-1=\underbrace{\overline{111\ldots11}}_{n-digits}\cdot\underbrace{\overline{9\underbrace{0\ldots0}_{n-1\,digits}9\underbrace{0\ldots0}_{n-1\,digits}9\underbrace{0\ldots0}_{n-1\,digits}9}}_{(k-1)n+1-digits}$$

$$10^{nk}=\underbrace{\overline{111\ldots11}}_{n-digits}\cdot\underbrace{\overline{9\underbrace{0\ldots0}_{n-1\,digits}9\underbrace{0\ldots0}_{n-1\,digits}9\underbrace{0\ldots0}_{n-1\,digits}9}}_{(k-1)n+1-digits}+1$$

$$10^{nk}=\underbrace{\overline{111\ldots11}}_{n-digits}\cdot9\cdot\underbrace{\overline{1\underbrace{0\ldots0}_{n-1\,digits}1\underbrace{0\ldots0}_{n-1\,digits}1\underbrace{0\ldots0}_{n-1\,digits}1}}_{(k-1)n+1-digits}+1$$

$$10^{nk}=\underbrace{\overline{999\ldots99}}_{n-digits}\cdot\underbrace{\overline{1\underbrace{0\ldots0}_{n-1\,digits}1\underbrace{0\ldots0}_{n-1\,digits}1\underbrace{0\ldots0}_{n-1\,digits}1}}_{(k-1)n+1-digits}+1$$

$$10^{nk}=\left(2\cdot\underbrace{\overline{499\ldots99}}_{n-digits}+1\right)\cdot\underbrace{\overline{1\underbrace{0\ldots0}_{n-1\,digits}1\underbrace{0\ldots0}_{n-1\,digits}1\underbrace{0\ldots0}_{n-1\,digits}1}}_{(k-1)n+1-digits}+1$$

$$10^{nk}=2\cdot\underbrace{\overline{499\ldots99}}_{n-digits}\cdot\underbrace{\overline{1\underbrace{0\ldots0}_{n-1\,digits}1\underbrace{0\ldots0}_{n-1\,digits}1\underbrace{0\ldots0}_{n-1\,digits}1}}_{(k-1)n+1-digits}+\underbrace{\overline{1\underbrace{0\ldots0}_{n-1\,digits}1\underbrace{0\ldots0}_{n-1\,digits}1\underbrace{0\ldots0}_{n-1\,digits}1}}_{(k-1)n+1-digits}+1$$

$$10^{nk}=\underbrace{\overline{499\ldots99}}_{n-digits}\cdot\underbrace{\overline{2\underbrace{0\ldots0}_{n-1\,digits}2\underbrace{0\ldots0}_{n-1\,digits}2\underbrace{0\ldots0}_{n-1\,digits}2}}_{(k-1)n+1-digits}+\underbrace{\overline{1\underbrace{0\ldots0}_{n-1\,digits}1\underbrace{0\ldots0}_{n-1\,digits}1\underbrace{0\ldots0}_{n-1\,digits}2}}_{(k-1)n+1-digits}$$

$$\underbrace{\overline{1\underbrace{0\ldots0}_{n-1\,digits}1\underbrace{0\ldots0}_{n-1\,digits}1\underbrace{0\ldots0}_{n-1\,digits}2}}_{(k-1)n+1-digits}<\underbrace{\overline{2\underbrace{0\ldots0}_{n-1\,digits}2\underbrace{0\ldots0}_{n-1\,digits}2\underbrace{0\ldots0}_{n-1\,digits}2}}_{(k-1)n+1-digits}$$

So, the extend of the division is $\underbrace{\overline{499\ldots99}}_{n-digits}$ and the rest is $\underbrace{\overline{2\underbrace{0\ldots0}_{n-1\,digits}2\underbrace{0\ldots0}_{n-1\,digits}2\underbrace{0\ldots0}_{n-1\,digits}2}}_{(k-1)n+1-digits}$.

Application 11. ***Find out the remainder of the division of the number 10^{nk}, $k\in\mathbb{N}^*$ to the number*** $\underbrace{\overline{\mathbf{4}\underbrace{\mathbf{0\ldots0}}_{n-1\,digits}\mathbf{4}\underbrace{\mathbf{0\ldots0}}_{n-1-digits}\mathbf{4}\underbrace{\mathbf{0\ldots0}}_{n-1-digits}\mathbf{4}}}_{(k-1)n+1-cifre}$.

Solution. Let be $\underbrace{\overline{9999\ldots999}}_{nk-digits}:\underbrace{\overline{111\ldots11}}_{n-digits}=\underbrace{\underbrace{\overline{9\underbrace{0\ldots0}_{n-1\,digits}9\underbrace{0\ldots0}_{n-1\,digits}9\underbrace{0\ldots0}_{n-1\,digits}9}}_{(k-1)-terms\ of\ \underbrace{\overline{0\ldots09}}_{n-\,digits}}}_{(k-1)n+1-digits}$

$$10^{nk}-1=\underbrace{\overline{111\ldots11}}_{n-digits}\cdot\underbrace{\overline{9\underbrace{0\ldots0}_{n-1\,digits}9\underbrace{0\ldots0}_{n-1\,digits}9\underbrace{0\ldots0}_{n-1\,digits}9}}_{(k-1)n+1-digits}$$

$$10^{nk}=\underbrace{\overline{111\dots11}}_{n-digits}\cdot\underbrace{\overline{9\underbrace{0\dots0}_{n-1\ digits}9\underbrace{0\dots0}_{n-1\ digits}9\underbrace{0\dots0}_{n-1\ digits}9}}_{(k-1)n+1-digits}+1$$

$$10^{nk}=\underbrace{\overline{111\dots11}}_{n-digits}\cdot9\cdot\underbrace{\overline{1\underbrace{0\dots0}_{n-1\ digits}1\underbrace{0\dots0}_{n-1\ digits}1\underbrace{0\dots0}_{n-1\ digits}1}}_{(k-1)n+1-digits}+1$$

$$10^{nk}=\underbrace{\overline{999\dots99}}_{n-digits}\cdot\underbrace{\overline{1\underbrace{0\dots0}_{n-1\ digits}1\underbrace{0\dots0}_{n-1\ digits}1\underbrace{0\dots0}_{n-1\ digits}1}}_{(k-1)n+1-digits}+1$$

$$10^{nk}=\left(4\cdot\underbrace{\overline{2499\dots99}}_{n-digits}+3\right)\cdot\underbrace{\overline{1\underbrace{0\dots0}_{n-1\ digits}1\underbrace{0\dots0}_{n-1\ digits}1\underbrace{0\dots0}_{n-1\ digits}1}}_{(k-1)n+1-digits}+1$$

$$10^{nk}=4\cdot\underbrace{\overline{2499\dots99}}_{n-digits}\cdot\underbrace{\overline{1\underbrace{0\dots0}_{n-1\ digits}1\underbrace{0\dots0}_{n-1\ digits}1\underbrace{0\dots0}_{n-1\ digits}1}}_{(k-1)n+1-digits}+$$

$$+3\cdot\underbrace{\overline{1\underbrace{0\dots0}_{n-1\ digits}1\underbrace{0\dots0}_{n-1\ digits}1\underbrace{0\dots0}_{n-1\ digits}1}}_{(k-1)n+1-digits}+1$$

$$10^{nk}=\underbrace{\overline{2499\dots99}}_{n-digits}\cdot\underbrace{\overline{4\underbrace{0\dots0}_{n-1\ digits}4\underbrace{0\dots0}_{n-1\ digits}4\underbrace{0\dots0}_{n-1\ digits}4}}_{(k-1)n+1-digits}+$$

$$+\underbrace{\overline{3\underbrace{0\dots0}_{n-1\ digits}3\underbrace{0\dots0}_{n-1\ digits}3\underbrace{0\dots0}_{n-1\ digits}4}}_{(k-1)n+1-digits}$$

But $\underbrace{\overline{3\underbrace{0\dots0}_{n-1\ digits}3\underbrace{0\dots0}_{n-1\ digits}3\underbrace{0\dots0}_{n-1\ digits}4}}_{(k-1)n+1-digits}<\underbrace{\overline{4\underbrace{0\dots0}_{n-1\ digits}4\underbrace{0\dots0}_{n-1\ digits}4\underbrace{0\dots0}_{n-1\ digits}4}}_{(k-1)n+1-digits}$, hence the extend of the division is $\underbrace{\overline{2499\dots99}}_{n-digits}$ and the rest is $\underbrace{\overline{3\underbrace{0\dots0}_{n-1\ digits}3\underbrace{0\dots0}_{n-1\ digits}3\underbrace{0\dots0}_{n-1\ digits}4}}_{(k-1)n+1-digits}$.

Proposed problems:

1. Find out the remainder of the division of the number 10^{2022} to the number $\underbrace{\overline{7\,007\,007\dots007}}_{2019-digits}$.

2. Find out the remainder of the division of the number 10^{2024} to the number $\underbrace{\overline{2022\,2022\dots2022}}_{2024-digits}$.

3. Find out the remainder of the division of the number $10^{3k}, k \in \mathbb{N}^*$ to the number

$\underbrace{\overline{14\,014\,014\ldots014}}_{3k-1-digits}$.

4. Find out the remainder of the division of the number $10^{3k}, k \in \mathbb{N}^*$ to the number

$\underbrace{\overline{20\,020\,020\ldots020}}_{3k-1-digits}$.

5. Find out the remainder of the division of the number $10^{4k}, k \in \mathbb{N}^*$ to the number

$\underbrace{\overline{2021\,2021\ldots2021}}_{4k-digits}$.

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