A SIMPLE PROOF FOR YOUNG'S INEQUALITY AND APPLICATIONS

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Abstract. In this paper is presented a simple proof for Young's inequality and a few applications.

YOUNG'S INEQUALITY ($n = 2$):
If $x_1, x_2 \geq 0; p, q > 1; \frac{1}{p} + \frac{1}{q} = 1$ then:

$$x_1 x_2 \leq \frac{x_1^p}{p} + \frac{x_2^q}{q} \tag{1}$$

Equality holds for $x_1^p = x_2^q$.

Proof.
Let be $f : (0, \infty) \to \mathbb{R}; f(x) = e^x$.

$$f'(x) = e^x; f''(x) = e^x > 0 \Rightarrow f \text{ convex } \Rightarrow$$

$$\Rightarrow f(\lambda_1 a + \lambda_2 b) \leq \lambda_1 f(a) + \lambda_2 f(b);$$

$$a, b > 0; \lambda_1, \lambda_2 > 0; \lambda_1 + \lambda_2 = 1$$

$$\text{For } \lambda_1 = \frac{1}{p}; \lambda_2 = \frac{1}{q} \Rightarrow \lambda_1 + \lambda_2 = \frac{1}{p} + \frac{1}{q} = 1$$

$$f\left(\frac{1}{p}a + \frac{1}{q}b\right) \leq \frac{1}{p}f(a) + \frac{1}{q}f(b)$$

$$e^{\frac{1}{p}a + \frac{1}{q}b} \leq \frac{1}{p}e^a + \frac{1}{q}e^b$$

$$e^{\frac{1}{p}a} \cdot e^{\frac{1}{q}b} \leq \frac{1}{p}e^a + \frac{1}{q}e^b$$

$$\text{For } x_1 = e^{\frac{1}{p}a}; x_2 = e^{\frac{1}{p}b} \Rightarrow \frac{a}{p} = \ln x_1; \frac{b}{q} = \ln x_2$$

$$a = p \ln x_1; b = q \ln x_2$$

$$e^{\frac{1}{p} \cdot p \ln x_1} \cdot e^{\frac{1}{q} \cdot q \ln x_2} \leq \frac{1}{p}e^a + \frac{1}{q}e^b$$

$$e^{\ln x_1} \cdot e^{\ln x_2} \leq \frac{1}{p}e^{p \ln x_1} + \frac{1}{q}e^{q \ln x_2}$$

$$x_1 \cdot x_2 \leq \frac{1}{p}(e^{\ln x_1})^p + \frac{1}{q}(e^{\ln x_2})^q$$

$$x_1 \cdot x_2 \leq \frac{1}{p} \cdot x_1^p + \frac{1}{q} \cdot x_2^q$$

If $x_1^p = x_2^q$ then:

$$\frac{1}{p}x_1^p + \frac{1}{q}x_2^q = \frac{1}{p}x_1^p + \frac{1}{q}x_1^p = \left(\frac{1}{p} + \frac{1}{q}\right)x_1^p =$$

$$= x_1^p = x_1^{p\cdot 1} = x_1^{p(\frac{1}{p}+\frac{1}{q})} = x_1^{1+\frac{p}{q}} =$$
$$= x_1 \cdot x_1^{\frac{p}{q}} = x_1 \cdot (x_1^p)^{\frac{1}{q}} = x_1 \cdot (x_1^q)^{\frac{1}{q}} = x_1 x_2$$

□

YOUNG INEQUALITY ($n = 3$)
If $x_1, x_2, x_3 \geq 0; p, q, r > 1; \frac{1}{p} + \frac{1}{q} + \frac{1}{r} = 1$ then:

$$x_1 x_2 x_3 \leq \frac{x_1^p}{p} + \frac{x_2^q}{q} + \frac{x_3^r}{r} \tag{2}$$

Equality holds for: $x_1^p = x_2^q = x_3^r$.

Proof.
Let be $f : (0, \infty) \to \mathbb{R}; f(x) = e^x; f'(x) = e^x;$

$$f''(x) = e^x > 0 \Rightarrow f \text{ convex } \Rightarrow$$
$$\Rightarrow f(\lambda_1 a + \lambda_2 b + \lambda_3 c) \leq \lambda_1 f(a) + \lambda_2 f(b) + \lambda_3 f(c)$$
$$a, b, c > 0; \lambda_1, \lambda_2, \lambda_3 > 0; \lambda_1 + \lambda_2 + \lambda_3 = 1$$
$$\text{For } \lambda_1 = \frac{1}{p}; \lambda_2 = \frac{1}{q}; \lambda_3 = \frac{1}{r} \Rightarrow \lambda_1 + \lambda_2 + \lambda_3 = \frac{1}{p} + \frac{1}{q} + \frac{1}{r} = 1$$
$$f\Big(\frac{1}{p}a + \frac{1}{q}b + \frac{1}{r}c\Big) \leq \frac{1}{p}f(a) + \frac{1}{q}f(b) + \frac{1}{r}f(c)$$
$$e^{\frac{1}{p}a+\frac{1}{q}b+\frac{1}{r}c} \leq \frac{1}{p}e^a + \frac{1}{q}e^b + \frac{1}{r}e^c$$
$$e^{\frac{1}{p}a} \cdot e^{\frac{1}{q}b} \cdot e^{\frac{1}{r}c} \leq \frac{1}{p}e^a + \frac{1}{q}e^b + \frac{1}{r}e^c$$
$$\text{For } x_1 = e^{\frac{1}{p}a}; x_2 = e^{\frac{1}{q}b}; x_3 =^{\frac{1}{r}c} \Rightarrow$$
$$\Rightarrow \frac{a}{p} = \ln x_1; \frac{b}{q} = \ln x_2; \frac{c}{r} = \ln x_3$$
$$a = p \ln x_1; b = q \ln x_2; c = r \ln x_3$$
$$e^{\frac{1}{p}\cdot p \ln x_1} \cdot e^{\frac{1}{q}\cdot q \ln x_2} \cdot e^{\frac{1}{r}\cdot r \ln x_3} \leq \frac{1}{p}e^{p \ln x_1} + \frac{1}{q}e^{q \ln x_2} + \frac{1}{r}e^{r \ln x_3}$$
$$e^{\ln x_1} \cdot e^{\ln x_2} \cdot e^{\ln x_3} \leq \frac{1}{p} \cdot (e^{\ln x_1})^p + \frac{1}{q} \cdot (e^{\ln x_2})^q + \frac{1}{r} \cdot (e^{\ln x_3})^r$$
$$x_1 \cdot x_2 \cdot x_3 \leq \frac{1}{p} \cdot x_1^p + \frac{1}{q} \cdot x_2^q + \frac{1}{r} \cdot x_3^r$$

If $x_1^p = x_2^q = x_3^r$ then:

$$\frac{1}{p}x_1^p + \frac{1}{q}x_2^q + \frac{1}{r}x_3^r = \frac{1}{p}x_1^p + \frac{1}{q}x_1^p + \frac{1}{r}x_1^p = 1$$
$$= \Big(\frac{1}{p} + \frac{1}{q} + \frac{1}{r}\Big)x_1^p = 1 \cdot x_1^p = x_1^{p\cdot 1} = x_1^{p(\frac{1}{p}+\frac{1}{q}+\frac{1}{r})} =$$
$$= x_1^{1+\frac{p}{q}+\frac{p}{r}} = x_1 \cdot x_1^{\frac{p}{q}} \cdot x_1^{\frac{p}{r}} = x_1 \cdot (x_1^p)^{\frac{1}{q}} \cdot (x_1^p)^{\frac{1}{r}} =$$
$$= x_1 \cdot (x_2^q)^{\frac{1}{q}} \cdot (x_3^r)^{\frac{1}{r}} = x_1 x_2 x_3$$

□

GENERAL YOUNG'S INEQUALITY

If $x_1, x_2, \ldots, x_n \geq 0; p_1, p_2, \ldots, p_n > 1; n \in \mathbb{N}; n \geq 2; \frac{1}{p_1} + \frac{1}{p_2} + \ldots + \frac{1}{p_n} = 1$ then:

$$x_1 x_2 \cdot \ldots \cdot x_n \leq \frac{x_1^{p_1}}{p_1} + \frac{x_2^{p_2}}{p_2} + \ldots + \frac{x_n^{p_n}}{p_n} \tag{3}$$

Equality holds for: $x_1^{p_1} = x_2^{p_2} = \ldots = x_n^{p_n}$.

Proof.
Let be $f : (0, \infty) \to \mathbb{R}; f(x) = e^x$

$$f'(x) = e^x; f''(x) = e^x > 0 \Rightarrow f \text{ convex } \Rightarrow$$

$$\Rightarrow f(\lambda_1 a_1 + \lambda_2 a_2 + \ldots + \lambda_n a_n) \leq \lambda_1 f(a_1) + \lambda_2 f(a_2) + \ldots + \lambda_n f(a_n)$$

$$a_1, a_2, \ldots, a_n > 0; \lambda_1, \lambda_2, \ldots, \lambda_n > 0; \lambda_1 + \lambda_2 + \ldots + \lambda_n = 1$$

$$\text{For } \lambda_1 = \frac{1}{p_1}; \lambda_2 = \frac{1}{p_2}; \ldots; \lambda_n = \frac{1}{p_n}$$

$$\lambda_1 + \lambda_2 + \ldots + \lambda_n = \frac{1}{p_1} + \frac{1}{p_2} + \ldots + \frac{1}{p_n}$$

$$f\Big(\frac{1}{p_1} a_1 + \frac{1}{p_2} a_2 + \ldots + \frac{1}{p_n} a_n\Big) \leq \frac{1}{p_1} f(a_1) + \frac{1}{p_2} f(a_2) + \ldots + \frac{1}{p_n} f(a_n)$$

$$e^{\frac{1}{p_1} a_1 + \frac{1}{p_2} a_2 + \ldots + \frac{1}{p_n} a_n} \leq \frac{1}{p_1} e^{a_1} + \frac{1}{p_2} e^{a_2} + \ldots + \frac{1}{p_n} e^{a_n}$$

$$e^{\frac{1}{p_1} a_1} \cdot e^{\frac{1}{p_2} a_2} \cdot \ldots \cdot e^{\frac{1}{p_n} a_n} \leq \frac{1}{p_1} e^{a_1} + \frac{1}{p_2} e^{a_2} + \ldots + \frac{1}{p_n} e^{a_n}$$

$$\text{For: } x_1 = e^{\frac{1}{p_1} a_1}; x_2 = e^{\frac{1}{p_2} a_2}; \ldots; x_n = e^{\frac{1}{p_n} a_n}$$

$$a_1 = p_1 \ln x_1; a_2 = p_2 \ln x_2; \ldots; a_n = p_n \ln x_n$$

$$e^{\frac{1}{p_1} p_1 \ln x_1} \cdot e^{\frac{1}{p_2} \cdot p_2 \ln x_2} \cdot \ldots \cdot e^{\frac{1}{p_n} \cdot p_n \ln x_n} \leq$$

$$\leq \frac{1}{p_1} e^{p_1 \ln x_1} + \frac{1}{p_2} e^{p_2 \ln x_2} + \ldots + \frac{1}{p_n} e^{p_n \ln x_n}$$

$$e^{\ln x_1} \cdot e^{\ln x_2} \cdot \ldots \cdot e^{\ln x_n} \leq \frac{1}{p_1} \cdot (e^{\ln x_1})^{p_1} + \frac{1}{p_2} \cdot (e^{\ln x_2})^{p_2} + \ldots + \frac{1}{p_n} (e^{\ln x_n})^{p_n}$$

$$x_1 \cdot x_2 \cdot \ldots \cdot x_n \leq \frac{1}{p_1} x_1^{p_1} + \frac{1}{p_2} x_2^{p_2} + \ldots + \frac{1}{p_n} x_n^{p_n}$$

If $x_1^{p_1} = x_2^{p_2} = \ldots = x_n^{p_n}$ then:

$$\frac{1}{p_1} x_1^{p_1} + \frac{1}{p_2} x_2^{p_2} + \ldots + \frac{1}{p_n} x_n^{p_n} =$$

$$= \frac{1}{p_1} x_1^{p_1} + \frac{1}{p_2} x_1^{p_1} + \ldots + \frac{1}{p_n} x_1^{p_1} = \Big(\frac{1}{p_1} + \frac{1}{p_2} + \ldots + \frac{1}{p_n}\Big) x_1^{p_1} =$$

$$= x_1^{p_1} = x_1^{p_1 \cdot 1} = x_1^{p_1 (\frac{1}{p_1} + \frac{1}{p_2} + \ldots + \frac{1}{p_n})} =$$

$$= x_1^1 \cdot x_1^{\frac{p_1}{p_2}} \cdot x_1^{\frac{p_1}{p_3}} \cdot \ldots \cdot x_1^{\frac{p_1}{p_n}} =$$

$$= x_1 \cdot (x_1^{p_1})^{\frac{1}{p_2}} \cdot (x_1^{p_1})^{\frac{1}{p_3}} \cdot \ldots \cdot (x_1^{p_1})^{\frac{1}{p_n}} =$$

$$= x_1 \cdot (x_2^{p_2})^{\frac{1}{p_2}} \cdot (x_3^{p_3})^{\frac{1}{p_3}} \cdot \ldots \cdot (x_n^{p_n})^{\frac{1}{p_n}} =$$

$$= x_1 \cdot x_2 \cdot \ldots \cdot x_n$$

□

Application 1.
If $x_1, x_2 \geq 0$ then:

$$x_1 x_2 \leq \frac{1}{2}(x_1^2 + x_2^2)$$

Proof.
We take in (1): $p = q = 2$. Equality holds for $x_1 = x_2$. □

Application 2.
If $x_1, x_2 \geq 0$ then:

$$3x_1 x_2 \leq x_1^3 + 2x_2\sqrt{x_2}$$

Proof.
We take in (1): $p = 3; q = \frac{3}{2}$. Equality holds for $x_1^3 = x_2^{\frac{3}{2}}$. □

Application 3.
If $x_1, x_2, x_3 \geq 0$ then:

$$x_1 x_2 x_3 \leq \frac{1}{3}(x_1^3 + x_2^3 + x_3^3)$$

Proof.
We take in (2): $p = q = r = 3$. Equality holds for $x_1 = x_2 = x_3$. □

Application 4.
If $x_1, x_2, x_3 \geq 0$ then:

$$x_1 x_2 x_3 \leq \frac{1}{2}x_1^2 + \frac{1}{3}x_2^3 + \frac{1}{6}x_3^6$$

Proof.
We take in (2): $p = 2; q = 3; r = 6$. Equality holds for $x_1^2 = x_2^3 = x_3^6$. □

Application 5.
If $x_1, x_2, \ldots, x_n \geq 0$ then:

$$x_1 x_2 \cdot \ldots \cdot x_n \leq \frac{1}{n}(x_1^n + x_2^n + \ldots + x_n^n); n \in \mathbb{N}; n \geq 2$$

Proof.
We take in (3): $p_1 = p_2 = \ldots = p_n = \frac{1}{n}$. Equality holds for $x_1 = x_2 = \ldots = x_n$. □

Application 6.
YOUNG'S INEQUALITY INTEGRAL FORM ($n = 2$)
If $f, g : [a, b] \to [0, \infty); a < b; f, g$ - continuous; $p, q > 1; \frac{1}{p} + \frac{1}{q} = 1$ then:

$$\int_a^b f(x)g(x)dx \leq \frac{1}{p}\int_a^b f^p(x)dx + \frac{1}{q}\int_a^b g^q(x)dx$$

Proof.
We take in (1): $x_1 = f(x); x_2 = g(x)$

$$f(x)g(x) \leq \frac{1}{p}f^p(x) + \frac{1}{q}g^q(x)$$

By integrating:

$$\int_a^b f(x)g(x)dx \leq \frac{1}{p}\int_a^b f^p(x)dx + \frac{1}{q}\int_a^b g^q(x)dx$$

Equality holds for: $f^p(x) = g^q(x); (\forall)x \in [a, b]$.

□

Application 7.
YOUNG'S INEQUALITY INTEGRAL FORM ($n = 3$)
If $f, g, h : [a, b] \to [0, \infty); a < b; f, g, h$ - continuous; $p, q, r > 1; \frac{1}{p} + \frac{1}{q} + \frac{1}{r} = 1$ then:

$$\int_a^b f(x)g(x)h(x)dx \leq \frac{1}{p}\int_a^b f^p(x)dx + \frac{1}{q}\int_a^b g^q(x)dx + \frac{1}{r}\int_a^b h^r(x)dx$$

Proof.
We take in (2): $x_1 = f(x); x_2 = g(x); x_3 = h(x)$

$$f(x)g(x)h(x) \leq \frac{1}{p}f^p(x) + \frac{1}{q}g^q(x) + \frac{1}{r}h^r(x)$$

By integrating:

$$\int_a^b f(x)g(x)h(x)dx \leq \frac{1}{p}\int_a^b f^p(x)dx + \frac{1}{q}\int_a^b g^q(x)dx + \frac{1}{r}\int_a^b h^r(x)dx$$

Equality holds for: $f^p(x) = g^q(x) = h^r(x); (\forall)x \in [a, b]$.

□

Application 8.
YOUNG'S INEQUALITY GENERAL INTEGRAL FORM
If $f_1, f_2, \ldots, f_n : [a, b] \to [0, \infty); a < b; f_1, f_2, \ldots f_n$ - continuous;
$p_1, p_2, \ldots, p_n > 1; \frac{1}{p_1} + \frac{1}{p_2} + \ldots + \frac{1}{p_n} = 1; n \in \mathbb{N}; n \geq 2$ then:

$$\int_a^b f_1(x)f_2(x)\cdot\ldots\cdot f_n(x)dx \leq \frac{1}{p_1}\int_a^b f_1^{p_1}(x)dx + \frac{1}{p_2}\int_a^b f_2^{p_2}(x)dx + \ldots + \frac{1}{p_n}\int_a^b f_n^{p_n}(x)dx$$

Proof.
We take in (3):

$$x_1 = f_1^{p_1}(x); x_2 = f_2^{p_2}(x); \ldots; x_n = f_n^{p_n}(x)$$

$$f_1(x)f_2(x)\cdot\ldots\cdot f_n(x) \leq \frac{1}{p_1}\cdot f_1^{p_1}(x) + \frac{1}{p_2}\cdot f_2^{p_2}(x) + \ldots + \frac{1}{p_n}\cdot f_n^{p_n}(x)$$

$$\int_a^b f_1(x)f_2(x)\cdot\ldots\cdot f_n(x)dx \leq \frac{1}{p_1}\int_a^b f_1^{p_1}(x)dx + \frac{1}{p_2}\int_a^b f_2^{p_2}(x)dx + \ldots + \frac{1}{p_n}\int_a^b f_n^{p_n}(x)dx$$

Equality holds for:

$$f_1^{p_1}(x) = f_2^{p_2}(x) = \ldots = f_n^{p_n}(x); (\forall)x \in [a, b]$$

□

Application 9.
If $x_1, x_2 \geq 0; a \in (0, \frac{\pi}{2})$ then:

$$x_1x_2 \leq \frac{1}{\sin^2 a}\cdot x_1^{\sin^2 a} + \frac{1}{\cos^2 a}\cdot x_2^{\cos^2 a}$$

Proof.
We take in (1): $p = \frac{1}{\sin^2 a}; q = \frac{1}{\cos^2 a}$
Obviously: $p, q > 1; \frac{1}{p} + \frac{1}{q} = 1$. Equality holds for: $x_1^{\sin^2 a} = x_2^{\cos^2 a}$

□

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