FAMOUS INEQUALITIES IN TRIANGLE REDESIGNED WITH CONWAY'S METHOD

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Abstract. In this paper is presented the Conway's method used for redesign famous inequalities in triangle as Euler, Mitrinovic, Leibniz, Ionescu-Weitzenbock, Leuenberger, Steining, Băndilă, Neuberg, Gordon, Goldner, Curry, Doucet, Gotman and Makowski.

Main results:

If $x, y, z \in \mathbb{R}$ are such that: $x + y > 0; y + z > 0; z + x > 0; xy + yz + zx > 0$ then:

1. $\sum_{cyc}(x+y)\sqrt{(x+z)(y+z)} \geq 4(xy+yz+zx)$
2. $(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x})^2 \geq 6\sqrt{3(xy+yz+zx)}$
3. $2(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x})\sqrt{xy+yz+zx} \leq 3\sqrt{3(x+y)(y+z)(z+x)}$
4. $8(x+y+z)(xy+yz+zx) \leq 9(x+y)(y+z)(z+x)$
5. $x+y+z \geq \sqrt{3(xy+yz+zx)}$
6. $\frac{1}{\sqrt{x+y}}+\frac{1}{\sqrt{y+z}}+\frac{1}{\sqrt{z+x}} \geq 2\sqrt{\frac{3(xy+yz+zx)}{(x+y)(y+z)(z+x)}}$
7. $\frac{1}{\sqrt{x+y}}+\frac{1}{\sqrt{y+z}}+\frac{1}{\sqrt{z+x}} \leq \frac{\sqrt{3}(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x})}{2\sqrt{xy+yz+zx}}$
8. $\frac{1}{\sqrt{(x+y)(x+z)}}+\frac{1}{\sqrt{(y+z)(y+x)}}+\frac{1}{\sqrt{(z+x)(z+y)}} \geq \frac{4(xy+yz+zx)}{(x+y)(y+z)(z+x)}$
9. $\frac{1}{\sqrt{(x+y)(x+z)}}+\frac{1}{\sqrt{(y+z)(y+x)}}+\frac{1}{\sqrt{(z+x)(z+y)}} \leq \frac{(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x})^2}{4(xy+yz+zx)}$
10. $\frac{1}{x+y}+\frac{1}{y+z}+\frac{1}{z+x} \geq \frac{\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}}{\sqrt{(x+y)(y+z)(z+x)}}$
11. $\frac{1}{x+y}+\frac{1}{y+z}+\frac{1}{z+x} \leq \frac{(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x})^2}{4(xy+yz+zx)}$
12. $\sqrt{\frac{x+y}{x+z}}+\sqrt{\frac{x+z}{x+y}} \leq \frac{(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x})\sqrt{(x+y)(y+z)(z+x)}}{2(xy+yz+zx)}$
13. $(x+y+z)(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x})^2 \geq 18(xy+yz+zx)$
14. $\sqrt{(x+y)(x+z)}+\sqrt{(y+z)(y+x)}+\sqrt{(z+x)(z+y)} \geq \frac{36(xy+yz+zx)}{(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x})^2}$
15. $\sqrt{(x+y)(x+z)}+\sqrt{(y+z)(y+x)}+\sqrt{(z+x)(z+y)} \leq \frac{9(x+y)(y+z)(z+x)}{4(xy+yz+zx)}$
16. $\sqrt{(x+y)(x+z)}+\sqrt{(y+z)(y+x)}+\sqrt{(z+x)(z+y)} \geq 2\sqrt{3(xy+yz+zx)}$
17. $x^2+y^2+z^2 \geq xy+yz+zx$
18. $(x+y)(x+z)+(y+z)(y+x)+(z+x)(z+y) \geq 4(xy+yz+zx)$
19. $\frac{9\sqrt{(x+y)(y+z)(z+x)}}{\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}} \geq 2\sqrt{3(xy+yz+zx)}$
20. $\sqrt{3}(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}) \leq 4\sqrt{\frac{(x+y)(y+z)(z+x)}{xy+yz+zx}}+\frac{2\sqrt{xy+yz+zx}}{\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}}$
21. $\sqrt{x+\frac{y+z}{4}}+\sqrt{y+\frac{z+x}{4}}+\sqrt{z+\frac{x+y}{4}} \geq \frac{9\sqrt{xy+yz+zx}}{\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}}$
22. $\sqrt{x+\frac{y+z}{4}}+\sqrt{y+\frac{z+x}{4}}+\sqrt{z+\frac{x+y}{4}} \leq \frac{9\sqrt{(x+y)(y+z)(z+x)}}{4\sqrt{xy+yz+zx}}$
23. $\frac{3\sqrt{xy+yz+zx}}{\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}} \leq \sqrt[4]{\frac{3}{4}(xy+yz+zx)}$

CONWAY'S METHOD

Let be $x, y, z \in \mathbb{R}$ such that $x+y>0; y+z>0; z+x>0; xy+yz+zx>0$.
Denote $a=\sqrt{x+y}, b=\sqrt{y+z}, c=\sqrt{z+x}$.
We will prove that a, b, c can be sides in a triangle ABC:

$$a+b>c \Leftrightarrow \sqrt{x+y}+\sqrt{y+z}>\sqrt{z+x}$$

$$x+y+y+z+2\sqrt{(x+y)(y+z)}>z+x$$

$$y+\sqrt{(x+y)(y+z)}>0$$

$$y+\sqrt{xz+yz+xy+y^2}>y+|y|\geq 0$$

Analogous: $b+c>a; c+a>b$.
The semiperimeter:

$$s=\frac{1}{2}(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x})$$

We will prove that area:

$$F=\frac{1}{2}\sqrt{xy+yz+zx}$$

$$s-a=\frac{1}{2}(\sqrt{x+y}+\sqrt{z+x}-\sqrt{y+z})$$

$$s-b=\frac{1}{2}(\sqrt{x+y}+\sqrt{y+z}-\sqrt{z+x})$$

$$s-c=\frac{1}{2}(\sqrt{y+z}+\sqrt{z+x}-\sqrt{x+y})$$

$$s(s-a)=\frac{1}{4}((\sqrt{x+y}+\sqrt{z+x})^2-(\sqrt{y+z})^2)=$$

$$=\frac{1}{4}(x+y+z+x+2\sqrt{(x+y)(z+x)}-y-z)=$$

$$=\frac{1}{4}(2\sqrt{(x+y)(z+x)}+2x)=\frac{1}{2}(\sqrt{(x+y)(z+x)}+x)$$

$$(s-b)(s-c)=\frac{1}{4}((\sqrt{y+z})^2-(\sqrt{x+y}-\sqrt{z+x})^2)$$

$$=\frac{1}{4}(y+z-x-y-z-x+2\sqrt{(x+y)(z+x)})=$$

$$=\frac{1}{2}(\sqrt{(x+y)(z+x)}-x)$$

$$F^2=s(s-a)(s-b)(s-c)=$$

$$=\frac{1}{4}((\sqrt{(x+y)(z+x)})^2-x^2)=$$

$$=\frac{1}{4}(x^2+xy+yz+zx-x^2)=\frac{1}{4}(xy+yz+zx)$$

$$F=\frac{1}{2}\sqrt{xy+yz+zx}$$

Let r, R be inradii and circumradii in ΔABC:

$$r=\frac{F}{s}=\frac{\sqrt{xy+yz+zx}}{\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}}$$

$$R=\frac{abc}{4F}=\frac{\sqrt{(x+y)(y+z)(z+x)}}{2\sqrt{xy+yz+zx}}$$

Let m_a, m_b, m_c be medians in ΔABC:

$$m_a^2 = \frac{1}{2}(b^2+c^2) - \frac{1}{4}a^2 = \frac{1}{2}(x+z+x+y) - \frac{1}{4}(y+z) =$$

$$= \frac{2(2x+y+z)-y-z}{4} = \frac{4x+y+z}{4}$$

Analogous:

$$m_b^2 = \frac{x+4y+z}{4}; m_c^2 = \frac{x+y+4z}{4}$$

Let h_a, h_b, h_c be altitudes in ΔABC:

$$h_a = \frac{2F}{a} = \sqrt{\frac{xy+yz+zx}{y+z}} = \sqrt{x + \frac{yz}{y+z}}$$

Analogous:

$$h_b = \sqrt{\frac{xy+yz+xz}{z+x}} = \sqrt{y + \frac{zx}{z+x}}$$

$$h_c = \sqrt{\frac{xy+yz+zx}{x+y}} = \sqrt{z + \frac{xy}{x+y}}$$

Proof of 1.
In ΔABC the following relationship holds:

$$R \geq 2r \text{ (Euler)}$$

Replace the values of R, r with Conway's substitutions:

$$\frac{\sqrt{(x+y)(y+z)(z+x)}}{2\sqrt{xy+yz+zx}} \geq \frac{2\sqrt{xy+yz+zx}}{\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}}$$

$$\sum_{cyc}(x+y)\sqrt{(z+x)(z+y)} \geq 4(xy+yz+zx)$$

□

Proof of 2.
In ΔABC the following relationship holds:

$$s \geq 3\sqrt{3}r \text{ (Mitrinovic)}$$

Replace the values of s, r with Conway's substitutions:

$$\frac{\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}}{2} \geq 3\sqrt{3} \cdot \frac{\sqrt{xy+yz+zx}}{\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}}$$

$$(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x})^2 \geq 6\sqrt{3(xy+yz+zx)}$$

□

Proof of 3.
In ΔABC the following relationship holds:

$$s \leq \frac{3\sqrt{3}}{2}R \text{ (Mitrinovic)}$$

Replace the values of s, R with Conway's substitutions:

$$\frac{\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}}{2} \leq \frac{3\sqrt{3}}{2} \cdot \frac{\sqrt{(x+y)(y+z)(z+x)}}{2\sqrt{xy+yz+zx}}$$

$$2(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x})\sqrt{xy+yz+zx} \leq 3\sqrt{3(x+y)(y+z)(z+x)}$$

□

Proof of 4.
In ΔABC the following relationship holds:

$$a^2+b^2+c^2 \leq 9R^2 \text{ (Leibniz)}$$

Replace the values of a, b, c, R with Conway's substitutions:

$$(\sqrt{x+y})^2+(\sqrt{y+z})^2+(\sqrt{z+x})^2 \leq 9 \cdot \frac{(x+y)(y+z)(z+x)}{4(xy+yz+zx)}$$

$$2(x+y+z) \leq \frac{9(x+y)(y+z)(z+x)}{4(xy+yz+zx)}$$

$$8(x+y+z)(xy+yz+zx) \leq 9(x+y)(y+z)(z+x)$$

□

Proof of 5.
In ΔABC the following relationship holds:

$$a^2+b^2+c^2 \geq 4\sqrt{3}F \text{ (Ionescu-Weitzenbock)}$$

Replace the values of a, b, c, F with Conway's substitutions:

$$(\sqrt{x+y})^2+(\sqrt{y+z})^2+(\sqrt{z+x})^2 \geq 4\sqrt{3} \cdot \frac{1}{2}\sqrt{xy+yz+zx}$$

$$2(x+y+z) \geq 2\sqrt{3(xy+yz+zx)}$$

$$x+y+z \geq \sqrt{3(xy+yz+zx)}$$

□

Proof of 6.
In ΔABC the following relationship holds:

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \geq \frac{\sqrt{3}}{R} \text{ (Leuenberger)}$$

Replace the values of a, b, c, R with Conway's substitutions:

$$\frac{1}{\sqrt{x+y}}+\frac{1}{\sqrt{y+z}}+\frac{1}{\sqrt{z+x}} \geq \sqrt{3} \cdot \frac{2\sqrt{xy+yz+zx}}{\sqrt{(x+y)(y+z)(z+x)}}$$

$$\frac{1}{\sqrt{x+y}}+\frac{1}{\sqrt{y+z}}+\frac{1}{\sqrt{z+x}} \geq 2\sqrt{\frac{3(xy+yz+zx)}{(x+y)(y+z)(z+x)}}$$

□

Proof of 7.
In ΔABC the following relationship holds:

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \leq \frac{\sqrt{3}}{2r} \text{ (Leuenberger)}$$

Replace the values of a, b, c, r with Conway's substitutions:

$$\frac{1}{\sqrt{x+y}}+\frac{1}{\sqrt{y+z}}+\frac{1}{\sqrt{z+x}} \leq \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}}{\sqrt{xy+yz+zx}}$$

□

Proof of 8.
In ΔABC the following relationship holds:

$$\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}\geq\frac{1}{R^2}\text{ (Leuenberger)}$$

Replace the values of a,b,c,R with Conway's substitutions:

$$\frac{1}{\sqrt{(x+y)(x+z)}}+\frac{1}{\sqrt{(y+z)(y+x)}}+\frac{1}{\sqrt{(z+x)(z+y)}}\geq\frac{4(xy+yz+zx)}{(x+y)(y+z)(z+x)}$$

□

Proof of 9.
In ΔABC the following relationship holds:

$$\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}\leq\frac{1}{4r^2}\text{ (Leuenberger)}$$

Replace the values of a,b,c with Conway's substitutions:

$$\frac{1}{\sqrt{(x+y)(x+z)}}+\frac{1}{\sqrt{(y+z)(y+x)}}+\frac{1}{\sqrt{(z+x)(z+y)}}\leq\frac{(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x})^2}{4(xy+yz+zx)}$$

□

Proof of 10.
In ΔABC the following relationship holds:

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\geq\frac{1}{2Rr}\text{ (Steinig)}$$

Replace the values of a,b,c,r,R with Conway's substitutions:

$$\frac{1}{x+y}+\frac{1}{y+z}+\frac{1}{z+x}\geq\frac{\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}}{\sqrt{(x+y)(y+z)(z+x)}}$$

□

Proof of 11.
In ΔABC the following relationship holds:

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\leq\frac{1}{4r^2}\text{ (Steinig)}$$

Replace the values of a,b,c,r,R with Conway's substitutions:

$$\frac{1}{x+y}+\frac{1}{y+z}+\frac{1}{z+x}\leq\frac{(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x})^2}{4(xy+yz+zx)}$$

□

Proof of 12.
In ΔABC the following relationship holds:

$$\frac{b}{c}+\frac{c}{b}\leq\frac{R}{r}\text{ (Băndilă)}$$

Replace the value of b,c,r,R with Conway's substitutions:

$$\sqrt{\frac{x+y}{x+z}}+\sqrt{\frac{x+z}{x+y}}\leq\frac{(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x})\sqrt{(x+y)(y+z)(z+x)}}{2(xy+yz+zx)}$$

□

Proof of 13.
In ΔABC the following relationship holds:

$$a^2+b^2+c^2 \geq 36r^2 \text{ (Neuberg)}$$

Replace the values of a,b,c,r with Conway's substitutions:

$$(x+y+z)(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x})^2 \geq 18(xy+yz+zx)$$

□

Proof of 14.
In ΔABC the following relationship holds:

$$ab+bc+ca \geq 36r^2 \text{ (Leuenberger)}$$

Replace the values of a,b,c,r with Conway's substitutions:

$$\sqrt{(x+y)(x+z)}+\sqrt{(y+z)(y+x)}+\sqrt{(z+x)(z+y)} \geq \frac{36(xy+yz+zx)}{(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x})^2}$$

□

Proof of 15.
In ΔABC the following relationship holds:

$$ab+bc+ca \leq 9R^2 \text{ (Leuenberger)}$$

Replace the values of a,b,c,R with Conway's substitutions:

$$\sqrt{(x+y)(x+z)}+\sqrt{(y+z)(y+x)}+\sqrt{(z+x)(z+y)} \leq \frac{9(x+y)(y+z)(z+x)}{4(xy+yz+zx)}$$

□

Proof of 16.
In ΔABC the following relationship holds:

$$ab+bc+ca \geq 4\sqrt{3}F \text{ (Gordon)}$$

Replace the values of a,b,c,F with Conway's substitutions:

$$\sqrt{(x+y)(x+z)}+\sqrt{(y+z)(y+x)}+\sqrt{(z+x)(z+y)} \geq 2\sqrt{3(xy+yz+zx)}$$

□

Proof of 17.
In ΔABC the following relationship holds:

$$a^4+b^4+c^4 \geq 16F^2 \text{ (Goldner)}$$

Replace the values of a,b,c,F with Conway's substitutions:

$$(\sqrt{x+y})^4+(\sqrt{y+z})^4+(\sqrt{z+x})^4 \geq 16\cdot\frac{1}{4}(xy+yz+zx)$$

$$(x+y)^2+(y+z)^2+(z+x)^2 \geq 4(xy+yz+zx)$$

$$x^2+y^2+z^2 \geq xy+yz+zx$$

□

Proof of 18.
In ΔABC the following relationship holds:

$$a^2b^2+b^2c^2+c^2a^2 \ge 16F^2 \text{ (Goldner)}$$

Replace the values of a, b, c, F with Conway's substitutions:

$$(x+y)(x+z)+(y+z)(y+x)+(z+x)(z+y) \ge 4(xy+yz+zx)$$

□

Proof of 19.
In ΔABC the following relationship holds:

$$\frac{9abc}{a+b+c} \ge 4\sqrt{3}F \text{ (Curry)}$$

Replace the values of a, b, c, F with Conway's substitutions:

$$\frac{9\sqrt{(x+y)(y+z)(z+x)}}{\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}} \ge 2\sqrt{3(xy+yz+zx)}$$

□

Proof of 20.
In ΔABC the following relationship holds:

$$s\sqrt{3} \le 4R+r \text{ (Doucet)}$$

Replace the values of s, R, r with Conway's substitutions:

$$\sqrt{3}(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}) \le 4\sqrt{\frac{(x+y)(y+z)(z+x)}{xy+yz+zx}}+\frac{2\sqrt{xy+yz+zx}}{\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}}$$

□

Proof of 21.
In ΔABC the following relationship holds

$$m_a+m_b+m_c \ge 9r \text{ (Gotman)}$$

Replace the values of m_a, m_b, m_c with Conway's substitutions:

$$\sqrt{x+\frac{y+z}{4}}+\sqrt{y+\frac{z+x}{4}}+\sqrt{z+\frac{x+y}{4}} \ge \frac{9\sqrt{xy+yz+zx}}{\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}}$$

□

Proof of 22.
In ΔABC the following relationship holds:

$$m_a+m_b+m_c \le \frac{9R}{2} \text{ (Gotman)}$$

Replace the values of m_a, m_b, m_c, R with Conway's substitutions:

$$\sqrt{x+\frac{y+z}{4}}+\sqrt{y+\frac{z+x}{4}}+\sqrt{z+\frac{x+y}{4}} \le \frac{9\sqrt{(x+y)(y+z)(z+x)}}{4\sqrt{xy+yz+zx}}$$

□

Proof of 23.
In ΔABC the following relationship holds:

$$\frac{3}{\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}} \leq \sqrt[4]{3F^2} \text{ (Makowski)}$$

Replace the values of h_a, h_b, h_c, F with Conway's substitutions:

$$\frac{3\sqrt{xy+yz+zx}}{\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}} \leq \sqrt[4]{\frac{3}{4}(xy+yz+zx)}$$

□

Observations:
Recall these famous inequalities in triangles: (a, b, c - sides; r, R - inradii, circumradii, F - area, h_a, h_b, h_c - altitudes; m_a, m_b, m_c - medians; s -semiperimeter)

1. $R \geq 2r$ (EULER)
2. $s \geq 3\sqrt{3}r$ (MITRINOVIC I)
3. $s \leq \frac{3\sqrt{3}}{2}R$ (MITRINOVICI II)
4. $a^2+b^2+c^2 \leq 9R^2$ (LEIBNIZ)
5. $a^2+b^2+c^2 \geq 4\sqrt{3}F$ (IONESCU-WEITZENBOCK)
6. $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \geq \frac{\sqrt{3}}{R}$ (LEUENBERGER I)
7. $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \leq \frac{\sqrt{3}}{2r}$ (LEUENBERGER II)
8. $\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca} \geq \frac{1}{R^2}$ (LEUENBERGER III)
9. $\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca} \leq \frac{1}{4r^2}$ (LEUENBERGER IV)
10. $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2} \geq \frac{1}{2Rr}$ (STEINIG I)
11. $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2} \leq \frac{1}{4r^2}$ (STEINIG II)
12. $\frac{b}{c}+\frac{c}{b} \leq \frac{R}{r}$ (BĂNDILĂ)
13. $a^2+b^2+c^2 \geq 36r^2$ (NEUBERG)
14. $ab+bc+ca \geq 36r^2$ (LEUENBERGER V)
15. $ab+bc+ca \leq 9R^2$ (LEUENBERGER VI)
16. $ab+bc+ca \geq 4\sqrt{3}F$ (GORDON)
17. $a^4+b^4+c^4 \geq 16F^2$ (GOLDNER I)
18. $a^2b^2+b^2c^2+c^2a^2 \geq 16F^2$ (GOLDNER II)
19. $\frac{9abc}{a+b+c} \geq 4F\sqrt{3}$ (CURRY)
20. $s\sqrt{3} \leq 4R+r$ (DOUCET)
21. $m_a+m_b+m_c \geq 9r$ (GOTMAN I)
22. $m_a+m_b+m_c \leq \frac{9R}{2}$ (GOTMAN II)
23. $\frac{3}{\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}} \leq \sqrt[4]{3F^2}$ (MAKOWSKI)

References

[1] Romanian Mathematical Magazine - Interactive Journal, www.ssmrmh.ro

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