A GEOMETRIC PROBABILITY PROBLEM

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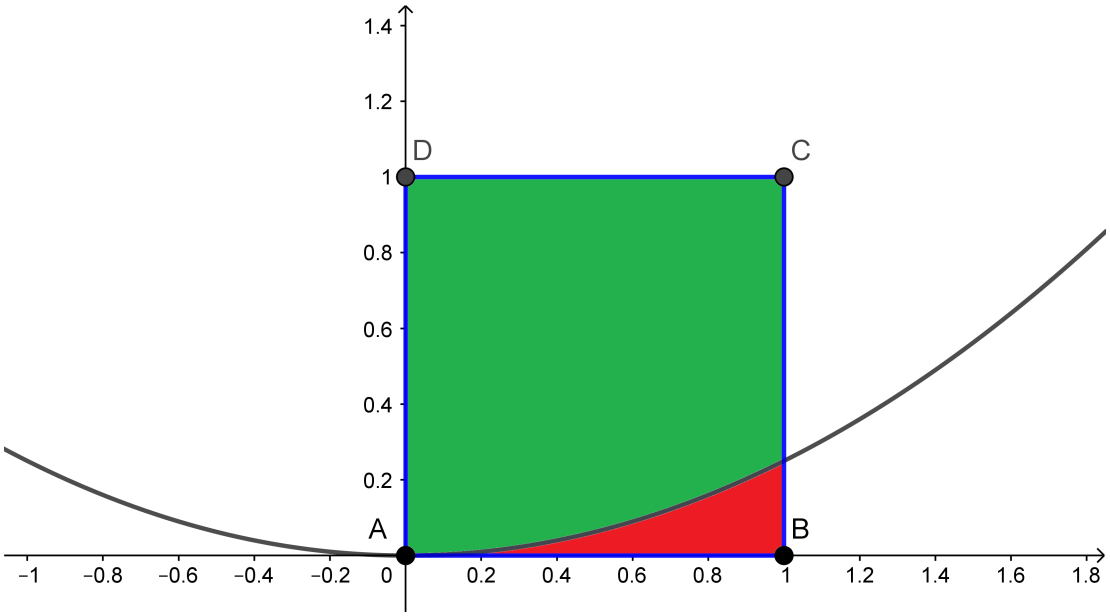
Choose a, b randomly in $[0,1]$. What is the probability that the equation $x^2 + ax + b = 0$ has no real solution?

Proof:



Sample space: Area of square ABCD: $S_{ABCD} = 1$.

For event $A = \{a \times b \in [0,1]^2 \ : \ \frac{a^2}{4} < b\}$, we have:

$$\mathbb{P}(A) = \frac{S_{\text{green}}}{S_{ABCD}} = 1 - \int_0^1 \frac{a^2}{4} da = 1 - \frac{1}{12} = \frac{11}{12}$$