TRIGONOMETRY INEQUALITY

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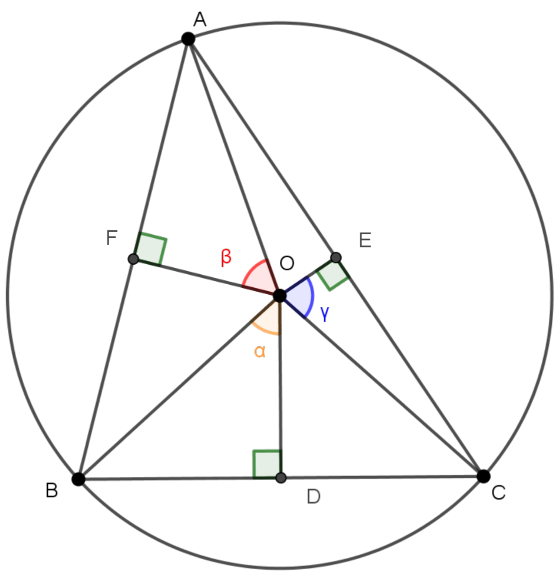
1 Introduction



Proof 1: Consider triangles $\Delta OBD, \Delta OAF, \Delta OCE$ respectively, we have:

$$\cos\alpha = \frac{OD}{R}$$
$$\cos\beta = \frac{OF}{R}$$
$$\cos\gamma = \frac{OE}{R}$$
$$\alpha + \beta + \gamma = \pi$$

Hence:

$$\cos\alpha + \cos\beta + \cos\gamma = \frac{OD + OF + OE}{R} \tag{1}$$

By Carnot's theorem and Euler's inequality we have:

$$OD + DE + OF = R + r \leq R + \frac{R}{2} = \frac{3R}{2} \tag{2}$$

From (1),(2) we have:

$$\cos\alpha + \cos\beta + \cos\gamma \leq \frac{3}{2}$$

Proof 2: Consider triangles $\Delta OBD, \Delta OAF, \Delta OCE$ respectively, we have:

$$\cos\alpha = \frac{OD}{R}$$
$$\cos\beta = \frac{OF}{R}$$
$$\cos\gamma = \frac{OE}{R}$$
$$\alpha + \beta + \gamma = \pi$$

Hence:

$$\cos\alpha + \cos\beta + \cos\gamma = \frac{OD + OF + OE}{R} \underbrace{\leq}_{\text{Erdos-Mordell}} \frac{OA + OB + OC}{2R} = \frac{3R}{2R} = \frac{3}{2}$$