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SPECIAL TRIANGLES

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We consider triangle ABC and X a point in the plane of triangle.

AX,BX,CX intersect BC,AC și CX in points X_a,X_b, X_c.

AX_a,BX_b,CX_c are cevians of point X.

AX_{a8}, BX_{b8}, CX_{c8} ↔ n_a , n_b, n_c are cevians of Nagel in triangle ABC.

Then we have the next theorem: n_a , n_b, n_c are cevians of Nagel in triangle ABC , can be lengths of sides of a triangle(Cevians as Sides of Triangles-Zvonko Cerin).

We show that: $p^2 = n_a^2 + 2r_a h_a$ (and analogs);

$r_a h_a = \frac{2r_a r_b r_c}{r_b+r_c}$ **(and analogs) ;**

$h_a = \frac{2r_b r_c}{r_b+r_c}$(and analogs);

$p^2 - n_a^2 = 2r_a h_a \rightarrow (p+n_a)(p-n_a) = 2r_a h_a \rightarrow p - n_a = \frac{2r_a h_a}{p+n_a} \rightarrow p = n_a + \frac{4r_a r_b r_c}{(n_a+p)(r_b+r_c)}$

p+p>p-true, cevians of Nagel can be lenghts of sides of a triangle then we have:

$n_a + n_b > n_c$(and analogs).**From all what we present we have the next conclusion:**

$\frac{1}{(n_a+p)(r_b+r_c)} + \frac{1}{(n_b+p)(r_a+r_c)} > \frac{1}{(n_c+p)(r_a+r_b)}$ (and analogs),**so**

1) $\frac{1}{(n_a+p)(r_b+r_c)}, \frac{1}{(n_b+p)(r_a+r_c)}, \frac{1}{(n_c+p)(r_a+r_b)}$ **can be lenghts of sides of a triangle.**

LEMA:If x,y,z are lengths sides of a triangle, then
$\sqrt{x}, \sqrt{y}, \sqrt{z}$ **are lengths sides of acute triangle.**

Now we will use the next relation relația $r_b + r_c = 4R\cos^2\frac{A}{2}$ (and analogs), and we obtain next results:

2) $\frac{1}{\sqrt{n_a+p}\cos\frac{A}{2}}, \frac{1}{\sqrt{n_b+p}\cos\frac{B}{2}}, \frac{1}{\sqrt{n_c+p}\cos\frac{C}{2}}$ **can be lenghts of sides of acute triangle.**

$\frac{2r_a h_a}{p+n_a} = p - n_a$ (and analogs) $\rightarrow \frac{2r_a h_a}{p-n_a} = p + n_a$ (and analogs).

n_a , n_b, n_c **are cevians of Nagel in triangle ABC , can be lengths of sides of a triangle, then we obtain:**

3) $\frac{r_a h_a}{p-n_a}, \frac{r_b h_b}{p-n_b}, \frac{r_c h_c}{p-n_c}$ can be lenghts of sides of a triangle.

4) $\frac{r_a h_a}{p+n_a}, \frac{r_b h_b}{p+n_b}, \frac{r_c h_c}{p+n_c}$ can be lenghts of sides of a triangle.

We use $r_a h_a = \frac{2r_a r_b r_c}{r_b+r_c}$ (and analogs) ,and we obtain:

5) $\frac{1}{(p-n_a)(r_b+r_c)}, \frac{1}{(p-n_b)(r_a+r_c)}, \frac{1}{(p-n_c)(r_a+r_b)}$ can be lenghts of sides of a triangle.

$r_b + r_c = 4R\cos^2\frac{A}{2}$ **(and analog) and we use Lema we obtain next results:**

6) $\frac{1}{\sqrt{p-n_a}\cos\frac{A}{2}}$, $\frac{1}{\sqrt{p-n_b}\cos\frac{B}{2}}$, $\frac{1}{\sqrt{p-n_c}\cos\frac{C}{2}}$ **can be lenghts of sides of acute triangle .**

We shown that $\frac{n_a}{h_a} = \frac{\sqrt{(b-c)^2+4r^2}}{2r}$ (and analogs), but also we shown that $\frac{n_a}{h_a}, \frac{n_b}{h_b}, \frac{n_c}{h_c}$ can be lenghts of sides of a triangle, so we will have the next results :

7) $\sqrt{(b-c)^2+4r^2}$,$\sqrt{(a-b)^2+4r^2}$, $\sqrt{(c-a)^2+4r^2}$ can be lenghts of sides of a triangle.

But AN_a=$\sqrt{(b-c)^2+4r^2}$(and analogs), N_a-point of Nagel ,so we obtain:

8) AN_a, BN_a, CN_a can be lenghts of sides of a triangle.

We know that AN_a=$\frac{an_a}{p}$ (and analogs)(RMM 33) and AG_e=$\frac{g_a(r_b+r_c)}{4R+r}$ (and analogs)(RMM 32) also we will use the next theorem:

(Murray Klamkin's Duality Principle for Triangle Inequalities):If P a point in Int(ΔABC), let PA=x,PB=y,PC=z,AB=c,AC=b,BC=a.Then ax,by,cz can be the lenghts of the sides of a triangle.

Useing this theorem we obtain the next results :

9)a^2n_a,b^2n_b, c^2n_c can be lenghts of sides of a triangle.

10) $a\sqrt{n_a}$, $b\sqrt{n_b}$,$c\sqrt{n_c}$ **can be lenghts of sides of acute triangle.**

11) $ag_a(r_b + r_c)$, $bg_b(r_a + r_c)$, $cg_c(r_a + r_b)$ **can be lenghts of sides of a triangle.**

12) $\sqrt{ag_a}\cos\frac{A}{2}$,$\sqrt{bg_b}\cos\frac{B}{2}$,$\sqrt{cg_c}\cos\frac{C}{2}$ **can be lenghts of sides of acute triangle.**

(we used relation $r_b + r_c = 4R\cos^2\frac{A}{2}$ (and analogs)**).**

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