ROMANIAN MATHEMATICAL MAGAZINE

Let x_1, x_2, x_3 be the roots of the equation $x^3 - 3x - 3 = 0$. Find:

$$\Omega = (x_1x_2 - x_3^2)(x_2x_3 - x_1^2)(x_1x_3 - x_2^2)$$

Proposed by Rovsen Pirguliyev-Azerbaijan

Solution 1 by Daniel Sitaru-Romania

$$\begin{cases} S_1 = x_1 + x_2 + x_3 = 0 \\ S_2 = x_1x_2 + x_2x_3 + x_3x_1 = -3 \\ S_3 = x_1x_2x_3 = 3 \end{cases}$$

$$x_1^3 = 3x_1 + 3, x_2^3 = 3x_2 + 3, x_3^3 = 3x_3 + 3$$

$$x_1^3 + x_2^3 + x_3^3 = 3x_1 + 3 + 3x_2 + 3 + 3x_3 + 3 = 3S_1 + 9 = 9$$

$$x_1^3x_2^3 + x_2^3x_3^3 + x_1^3x_3^3 =$$

$$= (3x_1 + 3)(3x_2 + 3) + (3x_2 + 3)(3x_3 + 3) + (3x_3 + 3)(3x_1 + 3) =$$

$$= 9S_2 + 18S_1 + 27 = 0$$

$$\Omega = (x_1x_2 - x_3^2)(x_2x_3 - x_1^2)(x_1x_3 - x_2^2) =$$

$$= (x_1x_2^2x_3 - x_1^3x_2 - x_2x_3^3 + x_1^2x_3^2)(x_1x_3 - x_2^2) =$$

$$= x_1^2x_2^2x_3^2 - x_1x_2^4x_3 - x_1^3x_2x_3 + x_1^3x_2^3 - x_1x_2x_3^4 + x_2^3x_3^3 - x_1^3x_3^3 - x_1^2x_2^2x_3^2 =$$

$$= -S_3(x_1^3 + x_2^3 + x_3^3) + (x_1^3x_2^3 + x_2^3x_3^3 + x_1^3x_3^3) = -27$$

Solution 2 by Ravi Prakash-New Delhi-India

$$x_1x_2x_3 = 3 \Rightarrow x_1x_2x_3\Omega = (x_1x_2x_3 - x_3^3)(x_1x_2x_3 - x_1^3)(x_1x_2x_3 - x_2^3)$$

$$x_1x_2x_3\Omega = (3 - x_3^3)(3 - x_1^3)(3 - x_2^3) \Rightarrow 3\Omega = (-3x_1)(-3x_2)(-3x_3)$$

$$3\Omega = -27x_1x_2x_3 \Rightarrow 3\Omega = -81 \Rightarrow \Omega = -27$$