ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z > 0$, then :

$$\sum_{cyc} x^8 z^4 \cdot \sum_{cyc} \frac{1}{(xy^2 + yz^2)^4} \geq \frac{9}{16}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\sum_{cyc} x^8 z^4 \cdot \sum_{cyc} \frac{1}{(xy^2 + yz^2)^4} = \sum_{cyc} x^4 y^8 \cdot \sum_{cyc} \frac{1^5}{(xy^2 + yz^2)^4} \overset{\text{Radon}}{\geq}$$

$$\sum_{cyc} x^4 y^8 \cdot \frac{3^5}{\left(\sum_{cyc} xy^2 + \sum_{cyc} yz^2\right)^4} = \frac{243\left(\sum_{cyc}\left(xy^2\right)^4\right)}{16\left(\sum_{cyc} xy^2\right)^4} \overset{\text{Holder}}{\geq} \frac{243 \cdot \frac{\left(\sum_{cyc} xy^2\right)^4}{27}}{16\left(\sum_{cyc} xy^2\right)^4} = \frac{9}{16},$$

$$\therefore \sum_{cyc} x^8 z^4 \cdot \sum_{cyc} \frac{1}{(xy^2 + yz^2)^4} \geq \frac{9}{16} \ \forall \ x, y, z > 0, '' ='' \text{ iff } x = y = z \ (\text{QED})$$