ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c,d>0$ such that $a+b+c+d=1$ then

$$\sum_{cyc}\frac{a}{a^3+b^4+c^4+d^4}\le\frac{1}{7abcd}$$

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By AM − GM inequality, we have $1=a+b+c+d\ge 4\sqrt[4]{abcd}$, then

$$abcd\le\frac{1}{4^4},\text{ and}$$

$$\frac{a^2bcd}{a^3+b^4+c^4+d^4}=\frac{a^2bcd}{4.\frac{a^3}{4}+b^4+c^4+d^4}\le\frac{a^2bcd}{7\sqrt[7]{\left(\frac{a^3}{4}\right)^4.b^4.c^4.d^4}}=\frac{\sqrt[7]{4^4a^2b^3c^3d^3}}{7}$$

$$\le\frac{\sqrt[7]{4^4\left(\frac{1}{4^4}\right)^2bcd}}{7}\le\frac{4.\frac{1}{4}+b+c+d}{7.7}=\frac{b+c+d+1}{49}.$$

Therefore

$$\sum_{cyc}\frac{a}{a^3+b^4+c^4+d^4}\le\sum_{cyc}\frac{b+c+d+1}{49abcd}=\frac{3(a+b+c+d)+4}{49abcd}=\frac{1}{7abcd}.$$