ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z > 0$ then:

$$\sum \frac{x^2 + 2\lambda + 1}{y + \lambda} \ge 6$$

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$$\frac{x^2 + 2\lambda + 1}{y + \lambda} = \frac{(x^2 + 1) + 2\lambda}{y + \lambda} \overset{AM-GM}{\ge} \frac{(2x) + 2\lambda}{y + \lambda} = \frac{2(x + \lambda)}{y + \lambda} \quad (1)$$

$$\sum \frac{x^2 + 2\lambda + 1}{y + \lambda} \overset{(1)}{\ge} \sum \frac{2(x + \lambda)}{y + \lambda} = 2\sum \frac{(x + \lambda)}{y + \lambda} \overset{AM-GM}{\ge}$$

$$\ge 6\sqrt[3]{\frac{(x + \lambda)}{y + \lambda}\,\frac{(y + \lambda)}{z + \lambda}\,\frac{(z + \lambda)}{x + \lambda}} = 6$$

$$\textit{Equality holds for } x = y = z$$