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If $a, b, c \geq 0$, $a + b + c = 2$ and $\lambda \geq 1$, then:

$$\sum_{cyc} \sqrt{\frac{b+c}{a^2+\lambda}} \geq 2\sqrt{\frac{2}{\lambda}}$$

Proposed by Marin Chirciu-Romania

Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

We have :

$$\begin{aligned} \sum_{cyc} \sqrt{\frac{b+c}{a^2+\lambda}} &= \sum_{cyc} \frac{b+c}{\sqrt{(a^2+\lambda)(2-a)}} = \sum_{cyc} \frac{b+c}{\sqrt{2\lambda - a[(a-1)^2 + (\lambda-1)]}} \geq \\ &\geq \sum_{cyc} \frac{b+c}{\sqrt{2\lambda}} \geq (a+b+c) \sqrt{\frac{2}{\lambda}} = 2\sqrt{\frac{2}{\lambda}} \end{aligned}$$

as desired. Equality holds iff $a = 2, b = c = 0$ and permutation.