ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$ then:

$$\sum \frac{a+b}{a^5+b^5+8} \le \frac{3}{5}$$

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Solution by Tapas Das-India

$$We\ will\ show\ a^5+b^5+8 \ge 5(a+b)\ (1)$$

$$Proof: we\ need\ to\ show\ a^5+b^5+8 \ge 5(a+b)\ or$$

$$\frac{(a+b)^5}{16}+8 \ge 5(a+b)\ (CBS)\ \ or$$

$$(a+b)^5+128 \ge 80(a+b) or$$

$$t^5-80t+128 \overset{a+b=t>0}{\ge}$$

$$(t-2)^2(t^3+4t^2+12t+32) \ge 0\ true\ as\ t > 0$$

$$\sum \frac{a+b}{a^5+b^5+8} \overset{(1)}{\le} \sum \frac{a+b}{5(a+b)} = \sum \frac{1}{5} = \frac{3}{5}$$

$$Equality\ holds\ for\ a=b=c$$