ROMANIAN MATHEMATICAL MAGAZINE

If $x,y,z,n>0$ and $\lambda>0$ then:

$$\sum\frac{x^2+n^2}{y+\lambda z}\geq\frac{6n}{\lambda+1}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\sum\frac{x^2+n^2}{y+\lambda z}=\sum\frac{x^2}{y+\lambda z}+\sum\frac{n^2}{y+\lambda z}\overset{Bergstrom}{\geq}$$

$$\geq\frac{(\sum x)^2}{(\sum x)(\lambda+1)}+n^2\frac{(1+1+1)^2}{(\sum x)(\lambda+1)}=$$

$$=\frac{(\sum x)^2}{(\sum x)(\lambda+1)}+n^2\frac{9}{(\sum x)(\lambda+1)}\overset{Am-Gm}{\geq}2\sqrt{\frac{(\sum x)^2}{(\sum x)(\lambda+1)}\cdot n^2\frac{9}{(\sum x)(\lambda+1)}}=\frac{6n}{\lambda+1}$$

Equality holds for $x=y=z=n$.