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If $a, b, c > 0$ and, $a + b + c = 3, \lambda \geq 0$ then:

$$\sum \frac{a^2}{\sqrt{a + \lambda b}} \geq \frac{3}{\sqrt{\lambda + 1}}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\begin{aligned} \sum \frac{a^2}{\sqrt{a + \lambda b}} &\stackrel{\text{Bergstrom}}{\geq} \frac{(a + b + c)^2}{\sum \sqrt{a + \lambda b}} \stackrel{\text{CBS}}{\geq} \\ &\frac{(a + b + c)^2}{\sqrt{3(a + b + c)(\lambda + 1)}} \stackrel{a+b+c=3}{=} \frac{9}{3} \frac{1}{\sqrt{\lambda + 1}} = \frac{3}{\sqrt{\lambda + 1}} \end{aligned}$$

Equality holds for $a = b = c = 1$