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If $a,b,c>0$, $a+b+c=3$, $\lambda\geq 0$ then:

$$\sum\frac{ab}{\sqrt{(\lambda+1)a^2-\lambda ab+(\lambda+1)b^2}}\leq\frac{3}{\sqrt{\lambda+2}}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$(\lambda+1)a^2-\lambda ab+(\lambda+1)b^2\overset{AM-GM}{\geq}2(\lambda+1)ab-\lambda ab=(\lambda+2)ab\ (1)$$

$$\sum\sqrt{ab}\overset{CBS}{\leq}\sqrt{3(ab+bc+ca)}\leq\sqrt{\frac{3(a+b+c)^2}{3}}=a+b+c=3\ (2)$$

$$\sum\frac{ab}{\sqrt{(\lambda+1)a^2-\lambda ab+(\lambda+1)b^2}}\overset{(1)}{\leq}\sum\frac{ab}{\sqrt{ab(\lambda+2)}}=\frac{1}{\sqrt{\lambda+2}}\sum\sqrt{ab}\overset{(2)}{\leq}\frac{3}{\sqrt{\lambda+2}}$$

Equality holds for a=b=c=1.