

ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z > 0$ then:

$$4 \sum x + \sum \frac{9}{1+x} \geq 24$$

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Solution by Tapas Das-India

$$\begin{aligned} 4 \sum x + \sum \frac{9}{1+x} &= 4 \sum (x+1) + \sum \frac{9}{1+x} - 12 \geq \\ &\stackrel{CBS}{\geq} 4 \sum (x+1) + 9 \frac{(1+1+1)^2}{\sum(x+1)} - 12 \stackrel{AM-GM}{\geq} \\ &\geq 2 \sqrt{4 \sum (x+1) \cdot 9 \frac{(3)^2}{\sum(x+1)}} - 12 = 2 \cdot 18 - 12 = 24 \end{aligned}$$

Equality holds for $x = y = z = \frac{1}{2}$.