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If $a_1, a_2, \dots, a_n > 0$, then :

$$\sum_{cyc} \frac{(a_1+a_2)^6}{(a_1a_2)^2} \geq 64\left(a_1^2+a_2^2+\cdots+a_n^2\right)$$

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$$\frac{(a_1+a_2)^6}{(a_1a_2)^2} = \frac{(a_1+a_2)^4(a_1+a_2)^2}{(a_1a_2)^2} = \frac{\left(a_1^2+a_2^2+2a_1a_2\right)^2(a_1+a_2)^2}{a_1^2a_2^2}$$

$$\overset{A-G}{\geq} \frac{4\left(a_1^2+a_2^2\right)(2a_1a_2)(4a_1a_2)}{a_1^2a_2^2} \Rightarrow \frac{(a_1+a_2)^6}{(a_1a_2)^2} \geq 32\left(a_1^2+a_2^2\right) \text{ and analogs}$$

$$\therefore \sum_{cyc} \frac{(a_1+a_2)^6}{(a_1a_2)^2} \geq 64\left(a_1^2+a_2^2+\cdots+a_n^2\right), '' ='' \text{ iff } a_1 = a_2 = \cdots = a_n \text{ (QED)}$$