ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0, a^2+b^2+c^2=3$ and $\lambda\geq 0$ then

$$\sum\frac{a^3}{a+\lambda}\geq\frac{3}{\lambda+1}$$

Proposed by Marin Chirciu – Romania

Solution by Amir Sofi – Kosovo

$$a,b,c>0,\lambda\geq 0, a^2+b^2+c^2=3$$

$$\frac{a^3}{a+\lambda}+\frac{b^3}{b+\lambda}+\frac{c^3}{c+\lambda}\geq\frac{3}{\lambda+1}$$

$$\frac{a^3}{a+\lambda}+\frac{b^3}{b+\lambda}+\frac{c^3}{c+\lambda}=\frac{a^4}{a^2+a\lambda}+\frac{b^4}{b^2+b\lambda}+\frac{c^4}{c^2+c\lambda}\geq$$

$$\geq\frac{(a^2+b^2+c^2)^2}{a^2+b^2+c^2+\lambda(a+b+c)}\geq\frac{(a^2+b^2+c^2)^2}{a^2+b^2+c^2+\lambda\sqrt{3(a^2+b^2+c^2)}}=$$

$$=\frac{3^2}{3+\lambda\sqrt{9}}=\frac{3}{\lambda+1}$$

Equality holds for $a=b=c=1$