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If $a,b,c>0, a+b+c=3$ and $\lambda\ge 0$ then:

$$\sum a\sqrt{2+\lambda b+\lambda c}\le 3\sqrt{2(\lambda+1)}$$

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Solution by Amir Sofi – Kosovo

$$a,b,c>0,\qquad a+b+c=3,\qquad \lambda\ge 0$$

$$a\sqrt{2+\lambda b+\lambda c}+b\sqrt{2+\lambda c+\lambda a}+c\sqrt{2+\lambda a+\lambda b}\le 3\sqrt{2(\lambda+1)}$$

$$a\sqrt{2+\lambda b+\lambda c}+b\sqrt{2+\lambda c+\lambda a}+c\sqrt{2+\lambda a+\lambda b}=$$

$$=\sqrt{a}\sqrt{2a+\lambda a(3-a)}+\sqrt{b}\sqrt{2b+\lambda b(3-b)}+\sqrt{c}\sqrt{2c+\lambda c(3-c)}\le$$

$$\le\sqrt{a+b+c}\sqrt{2(a+b+c)+\lambda\big(3(a+b+c)-(a^2+b^2+c^2)\big)}=$$

$$=\sqrt{3}\sqrt{6+\lambda\big(9-(a^2+b^2+c^2)\big)}\le\sqrt{3}\sqrt{6+\lambda\left(9-\frac{(a+b+c)^2}{3}\right)}=3\sqrt{2(\lambda+1)}$$

Equality holds for $a=b=c=1$.