If $a,b,c>0, a+b+c=3$ and $n,k\in\mathbb{N}$ then:

$$\sum\frac{a^{n+k}}{\sqrt{b+c}}\ge\frac{a^k+b^k+c^k}{\sqrt{2}}$$

Proposed by Marin Chirciu-Romania

Solution 1 by Tapas Das-India

$$\sum\frac{a^{m+k}}{\sqrt{b+c}}\underset{CBS}{\ge}\frac{\sum a^m\cdot\sum a^k}{3}$$

$$\sum\frac{a^{m+k}}{\sqrt{b+c}}\underset{Chebysev}{\ge}\frac{1}{3}\cdot\sum a^{m+k}\cdot\sum\frac{1}{\sqrt{b+c}}\underset{CBS}{\ge}\frac{1}{3}\cdot\frac{\sum a^m\cdot\sum a^k}{3}\cdot\frac{(1+1+1)^2}{\sqrt{6(a+b+c)}}\ge$$

$$\underset{CBS}{\ge}\frac{1}{9}\cdot\sum a^k\cdot\frac{1}{3^{m-1}}\cdot\left(\sum a\right)^m\cdot\frac{9}{\sqrt{6\times 3}}=\frac{1}{9}\cdot\sum a^k\cdot\frac{1}{3^{m-1}}\cdot(3)^m\cdot\frac{9}{3\sqrt{2}}$$

$$=\frac{1}{9}\cdot\sum a^k\cdot\frac{3\times 9}{3\sqrt{2}}=\frac{\sum a^k}{\sqrt{2}}$$

Solution 2 by Eric Cismaru-Romania

Without loss of generality, let us assume $a\ge b\ge c$.

This implies that $a^k\ge b^k\ge c^k$ and that $\frac{a^n}{\sqrt{b+c}}\ge\frac{b^n}{\sqrt{a+c}}\ge\frac{c^k}{\sqrt{a+b}}$. Applying now

Chebysev's Inequality for the sequences $\left\{\frac{a^n}{\sqrt{b+c}},\frac{b^n}{\sqrt{a+c}},\frac{c^n}{\sqrt{b+a}}\right\}$ and $\{a^k,b^k,c^k\}$, we obtain

$$\sum\frac{a^{n+k}}{\sqrt{b+c}}\ge\frac{1}{3}\left(\sum a^k\right)\left(\sum\frac{a^n}{\sqrt{b+c}}\right)$$

Using Holder's Inequality

$$\left(\sum\frac{a^n}{\sqrt{b+c}}\right)^{\frac{1}{n}}\cdot\left(\sum\sqrt{b+c}\right)^{\frac{1}{n}}\underbrace{(1+1+1)^{\frac{1}{n}}(1+1+1)^{\frac{1}{n}}\dots(1+1+1)^{\frac{1}{n}}}_{n-2\ times}\ge\sum a$$

and by raising this relationship to the power of n and dividing by 3^{n-2}, we find that

$$\sum\frac{a^n}{\sqrt{b+c}}\ge\frac{(\sum a)^n}{\left(\sum\sqrt{b+c}\right)\cdot 3^{n-2}}=\frac{3^n}{3^{n-2}\cdot\left(\sum\sqrt{b+c}\right)}$$

But $\sum\sqrt{b+c}\overset{C.B.S}{\le}\sqrt{6(a+b+c)}=3\sqrt{2}\Leftrightarrow\sum\frac{a^n}{\sqrt{b+c}}\ge\frac{3^n}{3^{n-1}.\sqrt{2}}=\frac{3}{\sqrt{2}}$, which is equivalent to

$$\sum\frac{a^{n+k}}{\sqrt{b+c}}\ge\left(\sum a^k\right)\cdot\frac{1}{3}\cdot\frac{3}{\sqrt{2}}=\frac{a^k+b^k+c^k}{\sqrt{2}}$$

In conclusion, $\sum \frac{a^{n+k}}{\sqrt{b+c}} \geq \frac{a^k+b^k+c^k}{\sqrt{2}}$

Equality holds when $a = b = c = 1$.