ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$ such that : $a^2 + b^2 + c^2 = 3$ and $\lambda \geq 0, n \geq 0$, then :

$$\frac{a^2}{b^2 + \lambda c + n} + \frac{b^2}{c^2 + \lambda a + n} + \frac{c^2}{a^2 + \lambda b + n} \geq \frac{3}{1 + \lambda + n}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\frac{a^2}{b^2 + \lambda c + n} + \frac{b^2}{c^2 + \lambda a + n} + \frac{c^2}{a^2 + \lambda b + n}$$

$$= \frac{a^4}{a^2b^2 + \lambda ca^2 + na^2} + \frac{b^4}{b^2c^2 + \lambda ab^2 + nb^2} + \frac{c^4}{c^2a^2 + \lambda bc^2 + nc^2} \overset{\text{Bergstrom}}{\geq}$$

$$\frac{\left(\sum_{cyc} a^2\right)^2}{\sum_{cyc} a^2b^2 + \lambda \sum_{cyc} ab^2 + n \sum_{cyc} a^2} \overset{\text{CBS}}{\geq} \frac{\left(\sum_{cyc} a^2\right)^2}{\sum_{cyc} a^2b^2 + \lambda . \sqrt{\left(\sum_{cyc} a^2b^2\right)\left(\sum_{cyc} a^2\right)} + n \sum_{cyc} a^2}$$

$$\geq \frac{\left(\sum_{cyc} a^2\right)^2}{\frac{1}{3}\left(\sum_{cyc} a^2\right)^2 + \lambda . \sqrt{\frac{1}{3}\left(\sum_{cyc} a^2\right)^2 . \left(\sum_{cyc} a^2\right)} + n \sum_{cyc} a^2}$$

$$\overset{a^2+b^2+c^2=3}{=} \frac{3\left(\sum_{cyc} a^2\right)}{\sum_{cyc} a^2 + \lambda . \sqrt{\frac{1}{3}\left(\sum_{cyc} a^2\right)^2 . 3} + n \sum_{cyc} a^2} = \frac{3}{1 + \lambda + n}$$

$$\therefore \frac{a^2}{b^2 + \lambda c + n} + \frac{b^2}{c^2 + \lambda a + n} + \frac{c^2}{a^2 + \lambda b + n} \geq \frac{3}{1 + \lambda + n}$$

$\forall\, a, b, c > 0 \,\middle|\, a^2 + b^2 + c^2 = 3$ and $\lambda \geq 0, n \geq 0$, " $=$ " iff $a = b = c = 1$ (QED)