ROMANIAN MATHEMATICAL MAGAZINE

If $a, b > 0, \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} = 2$ then:

$$\frac{1}{a^2 + b + 2b\sqrt{a}} + \frac{1}{b^2 + a + 2a\sqrt{b}} + \frac{\lambda}{a+b} \le \frac{\lambda + 1}{2}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} = 2 \ or, 1 \overset{AM-GM}{\le} \sqrt[4]{ab} \ or \ ab \ge 1$$

$$\frac{1}{a^2 + b + 2b\sqrt{a}} + \frac{1}{b^2 + a + 2a\sqrt{b}} + \frac{\lambda}{a+b} \overset{AM-GM}{\le}$$

$$\le \frac{2}{2a\sqrt{b} + 2b\sqrt{a}} + \frac{\lambda}{2\sqrt{ab}} = \frac{1}{(ab)\left(\frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}}\right)} + \frac{\lambda}{2\sqrt{ab}} \le \frac{\lambda + 1}{2},$$

$$since \ ab \ge 1 \ and \ \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} = 2$$