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If $a,b,c>0$ and $\lambda\geq\frac{1}{2}$ then:

$$\frac{a}{\sqrt{ab+\lambda b^2}}+\frac{b}{\sqrt{bc+\lambda c^2}}+\frac{c}{\sqrt{ca+\lambda a^2}}\geq\frac{3}{\sqrt{\lambda+1}}$$

Proposed by Marin Chirciu-Romania

Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

$$\sum_{cyc}\frac{a}{\sqrt{ab+\lambda b^2}}=2\sqrt{\lambda+1}.\sum_{cyc}\frac{a}{2\sqrt{b(\lambda+1).(a+\lambda b)}}\overset{AM-GM}{\geq}$$

$$\geq 2\sqrt{\lambda+1}.\sum_{cyc}\frac{a}{b(\lambda+1)+(a+\lambda b)}\geq$$

$$\overset{CBS}{\geq}2\sqrt{\lambda+1}.\frac{(a+b+c)^2}{\sum_{cyc}a[a+(1+2\lambda)b]}=2\sqrt{\lambda+1}.\frac{(a+b+c)^2}{\left(\sum_{cyc}a\right)^2+(2\lambda-1)\sum_{cyc}bc}\geq$$

$$\geq 2\sqrt{\lambda+1}.\frac{(a+b+c)^2}{\left(\sum_{cyc}a\right)^2+(2\lambda-1).\frac{\left(\sum_{cyc}a\right)^2}{3}}=\frac{3}{\sqrt{\lambda+1}}.$$

Equality holds iff $a=b=c$.