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If $a, b, c > 0$, $ab + bc + ca = 3abc$, $n \in \mathbb{N}$ then

$$\frac{1}{na+b} + \frac{1}{nb+c} + \frac{1}{nc+a} \leq \frac{3}{n+1}$$

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Solution by Tapas Das-India

$$\begin{aligned} \frac{1}{na+b} + \frac{1}{nb+c} + \frac{1}{nc+a} &\stackrel{AM-HM}{\leq} \frac{1}{(n+1)^2} \sum \left(\frac{n}{a} + \frac{1}{a} \right) = \\ &= \frac{1}{(n+1)^2} \left[\sum \frac{n}{a} + \sum \frac{1}{a} \right] = \frac{n+1}{(n+1)^2} \cdot \frac{ab+bc+ca}{abc} = \frac{3}{n+1} \end{aligned}$$

(since $ab + bc + ca = 3abc$)

Equality holds for $a = b = c = 1$.