ROMANIAN MATHEMATICAL MAGAZINE

If $a, b \geq \frac{1}{6}$ then:

$$\sqrt{8a^2+1}+\sqrt{8b^2+1}+\frac{32}{3}\left(\frac{1}{a+1}+\frac{1}{b+1}\right)\geq\frac{50}{3}$$

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$Lemma1$: For $x>0$, $\sqrt{8x^2+1}\geq\frac{8x+1}{3}$,

$Proof$: $\sqrt{8x^2+1}\geq\frac{8x+1}{3}$ or,

$$9(8x^2+1)\geq(8x+1)^2\ or,$$
$$8(x^2-2x+1)\geq 0\ or, (x-1)^2\geq 0\ (true)$$

$Lemma\ 2$: for $x>0$, $\frac{1+8x}{3}+\frac{32}{3}\frac{1}{x+1}\geq\frac{25}{3}$

$Proof$: $\frac{1+8x}{3}+\frac{32}{3}\frac{1}{x+1}\geq\frac{25}{3}$ or,

$$(1+8x)(x+1)+32\geq 25(x+1)or,$$

$$8(x^2-2x+1)\geq 0\ or, (x-1)^2\geq 0\ True$$

$$\sqrt{8a^2+1}+\sqrt{8b^2+1}+\frac{32}{3}\left(\frac{1}{a+1}+\frac{1}{b+1}\right)=$$

$$=\left[\sqrt{8a^2+1}+\frac{32}{3}\left(\frac{1}{a+1}\right)\right]+\left[\sqrt{8b^2+1}+\frac{32}{3}\left(\frac{1}{b+1}\right)\right]\overset{lemma1}{\geq}$$

$$\geq\left[\frac{8a+1}{3}+\frac{32}{3}\left(\frac{1}{a+1}\right)\right]+\left[\frac{8b+1}{3}+\frac{32}{3}\left(\frac{1}{b+1}\right)\right]\overset{lemma2}{\geq}$$

$$\geq\frac{25}{3}+\frac{25}{3}=\frac{50}{3}(Equality\ for\ a=b=1)$$