ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$, $a+b+c=3,\lambda\ge 0$ then:

$$\frac{a^3}{a^2+\lambda ab+b^2}+\frac{b^3}{b^2+\lambda bc+c^2}+\frac{c^3}{c^2+\lambda ca+a^2}\ge\frac{3}{\lambda+2}$$

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Solution by Tapas Das-India

$$\frac{a^3}{a^2+\lambda ab+b^2}=a-\frac{a(\lambda ab+b^2)}{a^2+\lambda\, ab+b^2}\overset{AM-GM}{\ge}a-\frac{ab(\lambda a+b)}{2ab+\lambda ab}=a-\frac{\lambda a+b}{\lambda+2}$$

$$\frac{a^3}{a^2+\lambda ab+b^2}+\frac{b^3}{b^2+\lambda bc+c^2}+\frac{c^3}{c^2+\lambda ca+a^2}\ge$$

$$\ge\sum\left[a-\frac{\lambda a+b}{\lambda+2}\right]=\sum a-\frac{(\lambda+1)(a+b+c)}{\lambda+2}=$$

$$=3-\frac{3\lambda+3}{\lambda+2}\left(as\ \sum a=3\right)=\frac{3}{\lambda+2}$$

$$Equality\ for\ \ a=b=c=1.$$