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If $a,b,c>0,\lambda\ge 2$ then:

$$\sum_{cyc}\frac{a}{\sqrt{\lambda a+b}}\le\sqrt{\frac{3(a+b+c)}{\lambda+1}}$$

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Solution by Tapas Das-India

$$\sum\frac{a}{\lambda a+b}=\frac{1}{\lambda}\sum\frac{a\lambda}{\lambda a+b}=\frac{1}{\lambda}\sum\left(1-\frac{b}{\lambda a+b}\right)=\frac{3}{\lambda}-\frac{1}{\lambda}\sum\frac{b^2}{\lambda ab+b^2}\overset{CBS}{\le}$$

$$\le\frac{3}{\lambda}-\frac{1}{\lambda}\frac{(a+b+c)^2}{(\sum\lambda ab)+\sum b^2}=\frac{3}{\lambda}-\frac{1}{\lambda}\frac{\sum a^2+2\sum ab}{(\sum\lambda ab)+\sum b^2}=$$

$$=\frac{2}{\lambda}+\frac{1}{\lambda}-\frac{1}{\lambda}\frac{\sum a^2+2\sum ab}{(\sum\lambda ab)+\sum b^2}=\frac{2}{\lambda}+\frac{1}{\lambda}\frac{\left((\lambda-2)\sum ab\right)}{\sum\lambda ab+\sum b^2}\overset{\sum a^2\ge\sum ab}{\le}$$

$$\le\frac{2}{\lambda}+\frac{1}{\lambda}\frac{\left((\lambda-2)\sum ab\right)}{\sum\lambda ab+\sum ab}=\frac{2}{\lambda}+\frac{1}{\lambda}\frac{\left((\lambda-2)\sum ab\right)}{(\lambda+1)\sum ab}=\frac{2}{\lambda}+\frac{1}{\lambda}\frac{(\lambda-2)}{(\lambda+1)}=\frac{3}{\lambda+1}\quad(1)$$

$$\sum\frac{a}{\sqrt{\lambda a+b}}=\sum\sqrt{a}.\sqrt{\frac{a}{\lambda a+b}}\overset{CBS}{\le}\sqrt{(a+b+c)\sum\frac{a}{\lambda a+b}}\overset{(1)}{\le}\sqrt{\frac{3(a+b+c)}{\lambda+1}}$$

$$Equality\ for\ a=b=c=1$$