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If $a, b, c > 0, abc \geq 1, \lambda \geq 0$ then:

$$\sum_{cyc} \frac{a^2}{bc + \lambda} \geq \frac{3}{\lambda + 1}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$ab + bc + ca \stackrel{AM-GM}{\geq} 3(abc)^{\frac{2}{3}} \geq 3(\text{since } abc \geq 1) \quad (1)$$

$$\begin{aligned} \sum \frac{a^2}{bc + \lambda} &\stackrel{\text{Bergstrom}}{\geq} \frac{(a+b+c)^2}{(\sum ab) + 3\lambda} \stackrel{(\sum ab) \leq \frac{(\sum a)^2}{3}}{\geq} \\ &\geq \frac{3 \sum ab}{\sum ab + 3\lambda} \stackrel{(1)}{\geq} \frac{3 \sum ab}{\sum ab + \lambda \sum ab} = \frac{3}{\lambda + 1} \end{aligned}$$

Equality holds for $a = b = c = 1$