

# ROMANIAN MATHEMATICAL MAGAZINE

If  $a, b, c > 0, a + b + c = 1, \lambda \geq 0$  then:

$$\sum_{cyc} \frac{a}{b(b + \lambda c)} \geq \frac{9}{\lambda + 1}$$

*Proposed by Marin Chirciu-Romania*

*Solution by Tapas Das-India*

$$\begin{aligned} \sum \frac{a}{b(b + \lambda c)} &= \sum \frac{\frac{a}{b}}{b + \lambda c} = \\ &= \sum \frac{\left(\sqrt{\frac{a}{b}}\right)^2}{b + \lambda c} \stackrel{\text{Bergstrom}}{\geq} \frac{\left(\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{c}} + \sqrt{\frac{c}{a}}\right)^2}{(\lambda + 1)(a + b + c)} \stackrel{\text{AM-GM}}{\geq} \\ &\geq \frac{(3)^2}{\lambda + 1} = \frac{9}{\lambda + 1} \text{ (since } a + b + c = 1) \end{aligned}$$

*Equality for  $a = b = c = \frac{1}{3}$*