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If $a,b,c>0,\lambda\geq 0$ then:

$$\sum_{cyc}\frac{bc}{a\sqrt{b+\lambda c}}\geq\frac{ab+bc+ca}{\sqrt{(\lambda+1)abc}}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\sum\frac{bc}{a\sqrt{b+\lambda c}}=\sum\frac{(bc)^{\frac{3}{2}}}{a\sqrt{bc}\sqrt{b+\lambda c}}=$$

$$=\frac{1}{\sqrt{abc}}\sum\frac{(bc)^{\frac{3}{2}}}{\sqrt{ab+\lambda ac}}\overset{Radon}{\geq}\frac{1}{\sqrt{abc}}\frac{(\sum bc)^{\frac{3}{2}}}{\sqrt{\sum ab+\lambda\sum ac}}=\frac{ab+bc+ca}{\sqrt{(\lambda+1)abc}}$$

$$Equality\ for\ a=b=c=1$$