If $x, y, z > 0, x + y + z = 1, \lambda \geq 1$ then:

$$\sum_{cyc} \frac{x^2 + \lambda xy}{x + y} \leq \frac{\lambda + 1}{2}$$

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Solution by Tapas Das-India

$$\sum \frac{x^2 + \lambda xy}{x + y} = \sum \left[x + \frac{(\lambda - 1)xy}{x + y} \right] =$$

$$= \sum x + (\lambda - 1) \sum \frac{xy}{x + y} \overset{Am-Gm}{\leq} 1 + (\lambda - 1) \sum \frac{xy}{2\sqrt{xy}} =$$

$$= 1 + \frac{(\lambda - 1)}{2} \sum \sqrt{xy} \overset{CBS}{\leq} 1 + \frac{(\lambda - 1)}{2} \sqrt{3(xy + yz + zx)} \leq$$

$$\leq 1 + \frac{(\lambda - 1)}{2} \sqrt{(x + y + z)^2} = 1 + \frac{(\lambda - 1)}{2} = \frac{\lambda + 1}{2}$$

$$Equality \; for \; x = y = z = \frac{1}{3}$$