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If $a, b, c > 0$, $abc = 1, n \in N$ then:

$$\sum_{cyc} \frac{(a+b)^{n+2}}{a^2+3} \geq 3 \cdot 2^n$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$We\ will\ show: 4\left(\sum a\right)^2 \geq 3\left(\sum a^2 + 9\right)\ (1)$$

$$proof: 4\left(\sum a\right)^2 \geq 3\left(\sum a^2 + 9\right)\ \ or\ \sum a^2 + 8\sum ab \geq 27$$

$$\sum a^2 + 8\sum ab \overset{Am-Gm}{\geq} 3(abc)^{\frac{2}{3}} + 24(abc)^{\frac{2}{3}} = 27\ (as\ abc = 1)$$

$$\left(\sum \frac{(a+b)^{n+2}}{a^2+3}\right)\left(\sum (a^2+3)\right)(1+1+1)^n \overset{Holder}{\geq} (2a+2b+2c)^{n+2}$$

$$or\left(\sum \frac{(a+b)^{n+2}}{a^2+3}\right) \geq \frac{(2a+2b+2c)^{n+2}}{(\sum(a^2+3))3^n} = \frac{2^n(a+b+c)^n}{3^n} \cdot \frac{4(a+b+c)^2}{a^2+b^2+c^2+9} \overset{Am-Gm}{\geq}$$

$$\geq \frac{2^n 3^n (abc)^{\frac{n}{3}}}{3^n} \frac{4(a+b+c)^2}{a^2+b^2+c^2+9} \overset{(1)\&abc=1}{\geq} 2^n \cdot 3$$

$$Equality\ for\ a = b = c = 1$$