ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ then:

$$\frac{a}{a+b}+\frac{b}{b+c}+\frac{c}{c+a}+\frac{9(a^2+b^2+c^2)}{8(a+b+c)^2}\geq\frac{15}{8}$$

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Solution by Tapas Das-India

$$\frac{a}{a+b}+\frac{b}{b+c}+\frac{c}{c+a}+\frac{9(a^2+b^2+c^2)}{8(a+b+c)^2}\geq\frac{15}{8}\ or$$

$$\sum\frac{a^2}{a^2+ab}+\frac{9(a^2+b^2+c^2)}{8(a+b+c)^2}\geq\frac{15}{8}\ or$$

$$\frac{(a+b+c)^2}{a^2+b^2+c^2+ab+bc+ca}+\frac{9(a^2+b^2+c^2)}{8(a+b+c)^2}\geq\frac{15}{8}or$$

$$\frac{\sum a^2+2\sum ab}{a^2+b^2+c^2+ab+bc+ca}+\frac{9(a^2+b^2+c^2)}{8(\sum a^2+2\sum ab)}\geq\frac{15}{8}\quad(1)$$

$$let\frac{a^2+b^2+c^2}{ab+bc+ca}=t\geq1\ \ then\ from(1)we\ get:$$

$$\frac{t+2}{t+1}+\frac{9t}{8(t+2)}\geq\frac{15}{8}\ or\ 8(t+2)^2+9t(t+1)\geq15(t+1)(t+2)$$

$$or\ 8(t^2+4t+4)+9t^2+9t\geq15(t^2+3t+2)or$$

$$2(t^2-2t+1)\geq0\ or\ (t-1)^2\geq0\ (true)$$

$$Equality\ for\ t=1\ or\ a^2+b^2+c^2=ab+bc+ca\ or\ a=b=c$$