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If $a,b,c,d>0$ and $\frac{1}{\sqrt{a+15}}+\frac{1}{\sqrt{b+15}}+\frac{1}{\sqrt{c+15}}+\frac{1}{\sqrt{d+15}}=1$ then:

$$3^a+3^b+3^c+3^d\ge 12$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$1=\frac{1^{\frac{3}{2}}}{\sqrt{a+15}}+\frac{1^{\frac{3}{2}}}{\sqrt{b+15}}+\frac{1^{\frac{3}{2}}}{\sqrt{c+15}}+\frac{1^{\frac{3}{2}}}{\sqrt{d+15}}\overset{Radon}{\ge}$$

$$\ge\frac{(4)^{\frac{3}{2}}}{(a+b+c+d+60)^{\frac{1}{2}}}$$

$$or(a+b+c+d+60)^{\frac{1}{2}}\ge(4)^{\frac{3}{2}}$$

$$or\ a+b+c+d+60\ge 64$$

$$or\ a+b+c+d\ge 4\ \ (1)$$

- $3^a+3^b+3^c+3^d\overset{AM-GM}{\ge}4\left(3^{a+b+c+d}\right)^{\frac{1}{4}}\overset{(1)}{\ge}4(3^4)^{\frac{1}{4}}=12$

- $Equality\ holds\ for\ a=b=c=d=1$