ROMANIAN MATHEMATICAL MAGAZINE

If $x,y,z>0$, $\sqrt{xy}+\sqrt{yz}+\sqrt{zx}=1$, $\lambda\ge 0$ then:

$$\sum\frac{x^3}{y(z+\lambda x)}\ge\frac{1}{\lambda+1}$$

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Solution by Tapas Das-India

$$\sqrt{xy}+\sqrt{yz}+\sqrt{zx}=1$$

$$\sqrt{3(xy+yz+zx)}\ge 1(CBS)$$

$$\sqrt{\frac{3(\sum x)^2}{3}}\ge 1\ or\ x+y+z\ge 1\ \ (1)$$

$$\sum\frac{x^3}{y(z+\lambda x)}\overset{Holder}{\ge}\frac{(x+y+z)^3}{3(\lambda+1)(xy+yz+zx)}\ge\frac{(x+y+z)^3}{\frac{3(\lambda+1)(x+y+z)^2}{3}}=\frac{x+y+z}{\lambda+1}\overset{(1)}{\ge}\frac{1}{\lambda+1}$$

$$Equality\ holds\ for\ x=y=z=\frac{1}{3}$$