ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0,\ \lambda \geq \frac{9}{8}$ then:

$$\frac{a}{a+b}+\frac{b}{b+c}+\frac{c}{c+a}+\frac{\lambda(a^2+b^2+c^2)}{(a+b+c)^2} \geq \ \lambda\frac{1}{3}+\frac{3}{2}$$

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Solution by Tapas Das-India

$$Let\ \frac{(a^2+b^2+c^2)}{ab+bc+ca}=t \geq 1$$

$$\frac{a}{a+b}+\frac{b}{b+c}+\frac{c}{c+a}+\frac{\lambda(a^2+b^2+c^2)}{(a+b+c)^2}=\sum\frac{a^2}{a^2+ab}+\frac{\lambda(a^2+b^2+c^2)}{(a+b+c)^2} \geq$$

$$\geq \frac{(a+b+c)^2}{a^2+b^2.+c^2+ab+bc+ca}+\frac{\lambda(a^2+b^2+c^2)}{(a+b+c)^2}=\frac{t+2}{t+1}+\frac{\lambda t}{t+2}$$

$$We\ \ need\ to\ show:\ \ \frac{t+2}{t+1}+\frac{\lambda t}{t+2} \geq \lambda\frac{1}{3}+\frac{3}{2}$$

$$t^2(4\lambda-3)-3t+6-4\lambda \geq 0\ or\ (t-1)[(4\lambda-3)(t+1)-3] \geq 0$$

$$true\ as\ \ t \geq 1\ and\ (4\lambda-3)(t+1) > \left(\frac{4.9}{8}-3\right)(1+1)=(1.5).3 > 3$$

$$since\ \lambda \geq \tfrac{9}{8}.\ Equality\ for\ a=b=c.$$