ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z, t > 0, x + y + z + t = 1$ *then:*

$$\sum \frac{1}{x^2 + x} \geq \frac{64}{5}$$

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Lemma:

$$\forall\, m > 0, \frac{1}{m^2 + m} \geq \frac{(176 - 384m)}{25},$$

Proof:

$$\frac{1}{m^2 + m} \geq \frac{(176 - 384m)}{25} \text{ or}$$

$$25 \geq -384m^3 - 384\, m^2 + 176m^2 + 176m \text{ or}$$

$$384m^3 + 208m^2 - 176m + 25 \geq 0 \text{ or}$$

$$6(64m^3) + 13(16m^2) - 44(4m) + 25 \geq 0 \text{ or}$$

$$6(4m)^3 + 13(4m)^2 - 44(4m) + 25 \geq 0 \text{ or}$$

$$6n^3 + 13n^2 - 44n + 25 \overset{4m=n>0}{\geq} 0 \text{ or}$$

$$(n - 1)^2(6n + 25) \geq 0 \text{ true}$$

$$\sum \frac{1}{x^2 + x} \overset{lemma}{\geq} \sum \frac{(176 - 384x)}{25} = \frac{(176).4 - 384(x + y + z + t)}{25} =$$

$$= \frac{704 - 384}{25} (as\ x + y + z + t = 1) = \frac{320}{25} = \frac{64}{5}$$

$$\text{Equality for } x = y = z = t = \frac{1}{4}$$