ROMANIAN MATHEMATICAL MAGAZINE

$$If\ a,b,c>0\ then$$

$$\sum\frac{\sqrt{ab}}{a+b}+\frac{1}{4}\sum\frac{a+b}{c}\geq 3$$

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Solution by Tapas Das-India

$$\sum\frac{\sqrt{ab}}{a+b}+\frac{1}{4}\sum\frac{a+b}{c}=$$

$$=\frac{\sqrt{ab}}{a+b}+\frac{\sqrt{bc}}{b+c}+\frac{\sqrt{ca}}{c+a}+\frac{1}{4}\frac{(a+b)}{c}+\frac{1}{4}\frac{(b+c)}{a}+\frac{1}{4}\frac{c+a}{b}\overset{Am-Gm}{\geq}$$

$$\geq\ 6\left(\frac{\sqrt{ab}}{a+b}\cdot\frac{\sqrt{bc}}{b+c}\cdot\frac{\sqrt{ca}}{c+a}\cdot\frac{1}{4}\frac{(a+b)}{c}\cdot\frac{1}{4}\frac{(b+c)}{a}\cdot\frac{1}{4}\frac{c+a}{b}\right)^{\frac{1}{6}}=$$

$$=6\left(\frac{abc(a+b)(b+c](c+a)}{64abc(a+b)(b+c)(c+a)}\right)^{\frac{1}{6}}=6.\left(\frac{1}{64}\right)^{\frac{1}{6}}=\frac{6}{2}=3$$

$$Equality\ \ for.a=b=c$$