ROMANIAN MATHEMATICAL MAGAZINE

*If* $a,b,c>0\ ,a^2+b^2+c^2=3,\lambda\geq 0$ *then*

$$\sum\frac{1}{a+\lambda b}\geq\frac{1}{(\lambda+1)(\sqrt{\lambda}+1)}\sum\sqrt{a^2+\lambda b^2}$$

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*Solution by Tapas Das-India*

$$(a+b+c)^2\overset{CBS}{\leq}3(a^2+b^2+c^2)=9\ (as\ ,a^2+b^2+c^2=3)or$$

$$(a+b+c)\leq 3\ (1)$$

$$R.H.S=\frac{1}{(\lambda+1)(\sqrt{\lambda}+1)}\sum\sqrt{a^2+\lambda b^2}\overset{CBS}{\leq}$$

$$\leq\frac{1}{(\lambda+1)(\sqrt{\lambda}+1)}\sqrt{3(a^2+\lambda b^2+b^2+\lambda c^2+c^2+\lambda a^2)}=$$

$$=\frac{1}{(\lambda+1)(\sqrt{\lambda}+1)}\sqrt{3(\lambda+1)(a^2+b^2+c^2)}=\frac{1}{(\lambda+1)(\sqrt{\lambda}+1)}\sqrt{3(\lambda+1)(3)}$$

$$=\frac{3}{\lambda+1}\ (since\ a^2+b^2+c^2=3)\ (2)$$

$$L.H.S=\sum\frac{1}{a+\lambda b}=\sum\frac{1^2}{a+\lambda b}\overset{Bergstrom}{\geq}\frac{(1+1+1)^2}{(a+b+c)(\lambda+1)}\geq$$

$$\overset{(1)}{\geq}\frac{9}{3(\lambda+1)}=\frac{3}{(\lambda+1)}\ (3)$$

$$from(2)\&(3)we\ can\ say\ L.H.S\geq R.H.S\ or$$

$$\sum\frac{1}{a+\lambda b}\geq\frac{1}{(\lambda+1)(\sqrt{\lambda}+1)}\sum\sqrt{a^2+\lambda b^2}$$

$$Equality\ for\ a=b=c=1$$